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Transformation of conformal coordinates of type Gauß-Krüger or UTM from a local datum (regional, National, European) to a global datum (WGS 84, ITRF 96)

Part II Case studies

Die Umformung großer Datenmengen konformer Koordinaten von einem lokalen in ein globales Datum bereitet Schwierigkeiten. Die Überlegungen zu dieser Thematik (behandelt im ersten Teil in AVN 4, 1998, S. 134) werden erweitert und fortgesetzt.

Introduction

A key problem of contemporary geodetic positioning is the transformation of mega data sets of *conformal coordinates* (*Gauß-Krüger* conformal coordinates, Universal Transverse Mercator (UTM), for instance) from a *local datum* to a *global datum* (*World Geodetic System 2000*, for instance). The satellite *Global Positioning System* (GPS: global problem solver) produces coordinates relating to the *International Reference Ellipsoid* $\mathbb{E}^2_{A,B}$ of semi-major axis A and semi-minor axis B (E. Grafarend and A. Ardalan 1999). In connection with a chart (*"map matching"*) GPS derived conformal coordinates of type *Gauß-Krüger* or *UTM* can only be used when they are transformed *from* the global datum *to* the local datum which the *chart* is based on. Here we continue the analysis and synthesis of such datum transformations of *Part* I (E. Grafarend and R. Syffus 1998) by means of *case studies.*

1 Datum transformation of conformal coordinates of type Gauß-Krüger/UTM

As outlined by means of the *commutative diagram of Figure 1*, Cartesian coordinates (X^1, X^2, X^3) (*capital letters: global datum*) are first transformed into Cartesian coordinates (x^1, x^2, x^3) (*small letters: local datum*) by means of the *conformal group* $\mathbb{C}_7(3)$, a transformation we also refer to as "rectangular datum transformation". In contrast, a transformation of ellipsoidal coordinates from (l, b, h) to (L, B, H) of type {ellipsoidal longitude, ellipsoidal latitude, ellipsoidal height} or from (L, B, H) to (l, b, h) is called a "*curvilinear datum transformation*" which

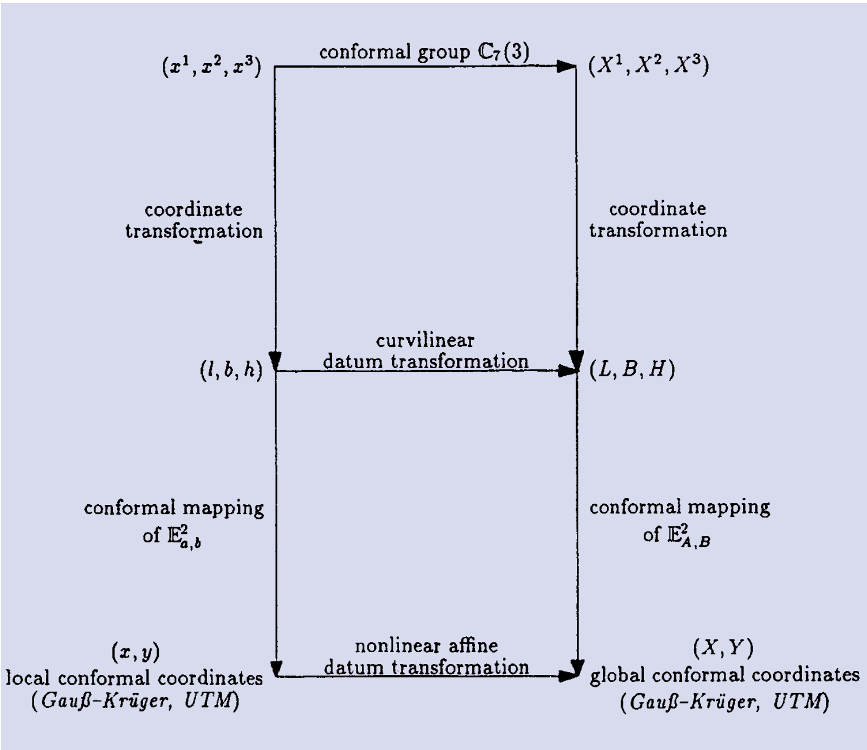

***Fig. 1:** Commutative diagram, rectangular, curvilinear and conformal datum transformation, three parameters of translation, three parameters of rotation, one scale parameter, local reference ellipsoid of resolution $\mathbb{E}^2_{a,b}$, global reference ellipsoid of revolution $\mathbb{E}^2_{A,B}$*

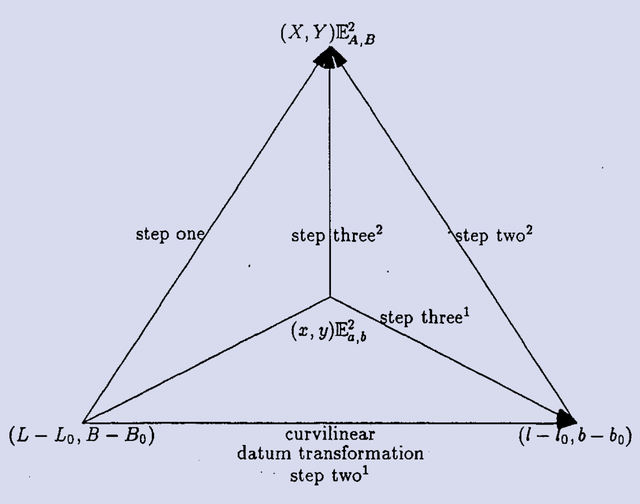

Fig. 2: Commutative diagram, genesis of the transformation of Cartesian conformal coordinates (x, y) in a local reference system to Cartesian conformal coordinates (X, Y) in a global reference system by means of a curvilinear datum transformation, three-step-procedure

is close to the identity. Based on this algorithm we presented in *Part* I (E. GRAFAREND and R. SYFFUS 1998) the *direct equations* of the datum transformation $(x, y) \to (X, Y)$ of conformal coordinates, or its *inverse equations* $(X, Y) \to (x, y)$. According to *Figure 2* the problem solution is devided in *three steps*:

In the *first step* we have to find a representation of (X,Y) as a function of $(L - L_0, B - B_0)$, namely $X\,(L - L_0, B - B_0)$, $Y\,(L - L_0, B - B_0)$, in general. The *second step* produces *at first* an equation to compute the "global" longitude, latitude, height relating to $\mathbb{E}^2_{A,B}$, namely $(l, b, h) \to (L, B, H)$: $L(l, b, h)$, $B(l, b, h)$, $H(l, b, h)$. *Secondly* we convert the equations of transformation to $X(l, b)$, $Y(l, b)$. In the *third step* the relation of (x, y) to (l, b) is found and finaly *the solution* of the transformation $(x, y) \to (X, Y)$ and, vice versa, $(X, Y) \to (x, y)$. A review of the polynomical representation of "*global*" conformal coordinates (X, Y) in terms of "*local*" conformal coordinates (x, y) and vice versa is given by *Box 1* and *Box 2*. Again we refer all details of such transformations to Part I (E. GRAFAREND and R. SYFFUS 1998).

Box 1: Polynomial representation of the global conformal coordinates(X,Y) in terms of local conformal coordinates (x,y) due to a curvilinear datum transformation, Gauß-Krüger conformal mapping or UTM, polynomial degree three, Easting X,x, Northing Y,y ("Rechtswert","Hochwert")

$$X = X(x,y,\rho,t_x,t_y,t_z,\alpha,\beta,\gamma,s,A,E^2,a,e^2) \tag{1.1}$$
$$= \rho[\bar{x}_{00} + \bar{x}_{10}\tfrac{x}{\rho} + \bar{x}_{01}(\tfrac{y}{\rho} - y_{00}) + \bar{x}_{20}(\tfrac{x}{\rho})^2 + \bar{x}_{11}\tfrac{x}{\rho}(\tfrac{y}{\rho} - y_{00}) + \bar{x}_{02}(\tfrac{y}{\rho} - y_{00})^2$$
$$+\bar{x}_{30}(\tfrac{x}{\rho})^3 + \bar{x}_{21}(\tfrac{x}{\rho})^2(\tfrac{y}{\rho} - y_{00}) + \bar{x}_{12}\tfrac{x}{\rho}(\tfrac{y}{\rho} - y_{00})^2 + \bar{x}_{03}(\tfrac{y}{\rho} - y_{00})^3] + O(4)$$

$$Y = Y(x,y,\rho,t_x,t_y,t_z,\alpha,\beta,\gamma,s,A,E^2,a,e^2) \tag{1.2}$$
$$= \rho[\bar{y}_{00} + \bar{y}_{10}\tfrac{x}{\rho} + \bar{y}_{01}(\tfrac{y}{\rho} - y_{00}) + \bar{y}_{20}(\tfrac{x}{\rho})^2 + \bar{y}_{11}\tfrac{x}{\rho}(\tfrac{y}{\rho} - y_{00}) + \bar{y}_{02}(\tfrac{y}{\rho} - y_{00})^2$$
$$+\bar{y}_{30}(\tfrac{x}{\rho})^3 + \bar{y}_{21}(\tfrac{x}{\rho})^2(\tfrac{y}{\rho} - y_{00}) + \bar{y}_{12}\tfrac{x}{\rho}(\tfrac{y}{\rho} - y_{00})^2 + \bar{y}_{03}(\tfrac{y}{\rho} - y_{00})^3] + O(4)$$

Box 2: Polynomial representation of local conformal coordinates(x,y) in terms of global conformal coordinates (X,Y) due to a curvilinear datum transformation, Gauß-Krüger conformal mapping or UTM, polynomial degree three, Easting X,x, Northing Y,y ("Rechtswert","Hochwert")

$$x = x(X,Y,\rho,t_x,t_y,t_z,\alpha,\beta,\gamma,s,A,E^2,a,e^2) \tag{1.3}$$
$$= \rho[x^{10}(\tfrac{X}{\rho} - \bar{x}_{00}) + x^{01}(\tfrac{Y}{\rho} - \bar{y}_{00}) + x^{20}(\tfrac{X}{\rho} - \bar{x}_{00})^2 + x^{11}(\tfrac{X}{\rho} - \bar{x}_{00})(\tfrac{Y}{\rho} - \bar{y}_{00})$$
$$+x^{02}(\tfrac{Y}{\rho} - \bar{y}_{00})^2 + x^{30}(\tfrac{X}{\rho} - \bar{x}_{00})^3 + x^{21}(\tfrac{X}{\rho} - \bar{x}_{00})^2(\tfrac{Y}{\rho} - \bar{y}_{00})$$
$$+x^{12}(\tfrac{X}{\rho} - \bar{x}_{00})(\tfrac{Y}{\rho} - \bar{y}_{00})^2 + x^{03}(\tfrac{Y}{\rho} - \bar{y}_{00})^3] + O(4)$$

$$y = y(X,Y,\rho,t_x,t_y,t_z,\alpha,\beta,\gamma,s,A,E^2,a,e^2) \tag{1.4}$$
$$= \rho[y_{00} + y^{10}(\tfrac{X}{\rho} - \bar{x}_{00}) + y^{01}(\tfrac{Y}{\rho} - \bar{y}_{00}) + y^{20}(\tfrac{X}{\rho} - \bar{x}_{00})^2 + y^{11}(\tfrac{X}{\rho} - \bar{x}_{00})(\tfrac{Y}{\rho} - \bar{y}_{00})$$
$$+y^{02}(\tfrac{Y}{\rho} - \bar{y}_{00})^2 + y^{30}(\tfrac{X}{\rho} - \bar{x}_{00})^3 + y^{21}(\tfrac{X}{\rho} - \bar{x}_{00})^2(\tfrac{Y}{\rho} - \bar{y}_{00})$$
$$+y^{12}(\tfrac{X}{\rho} - \bar{x}_{00})(\tfrac{Y}{\rho} - \bar{y}_{00})^2 + y^{03}(\tfrac{Y}{\rho} - \bar{y}_{00})^3] + O(4)$$

Some remarks to the possible *linearization* of the semi-major axis A = a + δa *and* the squared relative eccentricity $E^2 = e^2 + \delta e^2$ have to be made. At an early stage of the development of our algorithm we were not convinced whether the common linearization of the transformation equations with respect to the *form parameters* (A, E^2) or (a, e^2) would be numerically useful. Accordingly in a "*Studienarbeit*" D. FRIEDRICH (1998) compared the transformation (*i*) $(L, B, H) \rightarrow (X^1, X^2, X^3)$, (*ii*) $(X^1, X^2, X^3) \rightarrow (x^1, x^2, x^3)$, (*iii*) $(x^1, x^2, x^3) \rightarrow (l, b, h)$ *linearized in the form parameter variations* with the closed form equations: The "rectangular datum transformation" (*ii*) nearly produced identical results with and without linearization. In contrast, the "curvilinear datum transformation" {l(L, B, H), b(L, B, H), h(l, B, H)} was strongly affected: The systematic differences were *10 times larger*. Accordingly we decided to apply those "curvilinear datum transformation" *without* any linearization in the *form parameters* (A, E^2) and (a, e^2) respectively.

2 Numerical results

In our case studies we concentrated to the *State of Baden-Württemberg*. The transformation of 50 BWREF points from a global to a lokal datum and *vice versa* has been computed. *Table 2.1* summarizes those datum transformation parameters available to us.

Table 2.1: Datum transformation parameters global (WGS 84) to local (BW)

$$t_x = 592.270898m$$
$$t_y = 76.285723m$$
$$t_z = 407.334716m$$
$$\alpha = -1.092843''$$
$$\beta = -0.097832''$$
$$\gamma = 1.604106''$$
$$s = 8.537829ppm$$
$$a = 6377397.155076m$$
$$e^2 = 0.006674372231$$
$$A = 6378137.000m$$
$$E^2 = 0.00669437999$$

For space reasons we review the results for only *ten points*, both for the forward and backward transformations. *Table 2.2* and *Table 2.3* represent the differences between the Gauß-Krüger conformal coordinates (X, Y) and those computed ones $\{X(trans), Y(trans)\}$. Indeed the differences of the *Easting* ("*Rechtswert*") were larger than those of the *Northing* ("*Hochwert*"). We have to mention that all transformation parameters were based on those data of "*Deutsches Hauptdreiecksnetz*" (DHDN). Accordingly the accuracy of the transformation cannot be better than a few centimeters. For a more detailed analysis we have chosen *five points* (Katzenbuck, Gerabronn, Karlsruhe, Stuttgart, Oberkochen) whose *Gauß-Krüger conformal coordinates* as well as *ellipsoidal heights* are given in *Table 2.4*. *Table 2.5* and *Table 2.6* summarize those polynomial coefficients given in *Box 1* and *Box 2* representing $X = X(x, y, \rho, t_x, t_y, t_z, \alpha, \beta, \gamma, s, A, E^2, a, e^2)$, $Y = Y(x, y, \rho, t_x, t_y, t_z, \alpha, \beta, \gamma, s, A, E^2, a, e^2)$ and $x = x(X, Y, \rho, t_x, t_y, t_z, \alpha, \beta, \gamma, s, A, E^2, a, e^2)$, $y = y(X, Y, \rho, t_x, t_y, t_z, \alpha, \beta, \gamma, s, A, E^2, a, e^2)$, respectively.

Table 2.2: Difference between Gauß-Krüger conformal coordinates X and X_{trans}: "Easting"

pointnumber	X[m]	X_{trans}[m]	dX[mm]
6324	3558357.7304	3558357.7333	-2.9
6417	3473105.6664	3473105.6701	-3.7
6520	3503525.3824	3503525.3858	-3.4
6725	3567188.4423	3567188.4454	-3.1
6922	3529538.2613	3529538.2647	-3.4
7016	3462353.7891	3462353.7930	-3.9
7220	3506195.9031	3506195.9068	-3.7
7226	3579947.1053	3579947.1084	-3.1
7316	3462442.3184	3462442.3224	-4.0
7324	3556797.2523	3556797.2556	-3.3

Table 2.3: Difference between Gauß-Krüger conformal coordinates Y and Y_{trans}: "Northing"

pointnumber	Y[m]	Y_{Tra}[m]	dY[mm]
6324	5502059.3111	5502059.3116	-0.5
6417	5488314.3903	5488314.3904	-0.1
6520	5481082.8905	5481082.8908	-0.3
6725	5458730.7146	5458730.7152	-0.6
6922	5437066.5236	5437066.5240	-0.4
7016	5429412.0806	5429412.0807	-0.1
7220	5405925.8183	5405925.8187	-0.4
7226	5406962.3048	5406962.3055	-0.7
7316	5386837.0856	5386837.0857	-0.1
7324	5387475.3472	5387475.3477	-0.5

Table 2.4: Selected BW-points, Gauß-Krüger conformal coordinates, Easting ("Rechtswert") x, Northing ("Hochwert") y, ellipsoidal height h, name of the point

pointnumber	x[m]	y[m]	h[m]	name
6520	3503600.491	5480643.197	514.164	Katzenbuck
6725	3567263.651	5458291.202	477.449	Gerabronn
7016	3462429.201	5428972.406	277.644	Karlsruhe
7220	3506271.260	5405486.180	519.481	Stuttgart
7226	3580022.573	5406522.794	734.318	Oberkochen

Table 2.5: Transformation from a local to a global reference system, polynomial coefficients

Punkt 6520	$\bar{a}_{00}$	-75.66265	$\bar{b}_{00}$	5485113.89500
	$\bar{a}_{10}$	3601.00945	$\bar{b}_{10}$	-0.04992
	$\bar{a}_{01}$	-0.05588	$\bar{b}_{01}$	-4030.98709
	$\bar{a}_{20}$	-0.00001	$\bar{b}_{20}$	0.00002
	$\bar{a}_{11}$	-0.00013	$\bar{b}_{11}$	0.00003
	$\bar{a}_{02}$	0.00002	$\bar{b}_{02}$	0.00004
	$\bar{a}_{30}$	0.00000	$\bar{b}_{30}$	0.00000
	$\bar{a}_{21}$	0.00000	$\bar{b}_{21}$	0.00000
	$\bar{a}_{12}$	0.00000	$\bar{b}_{12}$	0.00000
	$\bar{a}_{03}$	0.00000	$\bar{b}_{03}$	0.00000

Fortsetzung Tab. 2.5

Punkt 6725	$\bar{a}_{00}$	-84.78176	$\bar{b}_{00}$	5462873.73554
	$\bar{a}_{10}$	67273.28296	$\bar{b}_{10}$	-1.03759
	$\bar{a}_{01}$	-0.06389	$\bar{b}_{01}$	-4142.01666
	$\bar{a}_{20}$	-0.00471	$\bar{b}_{20}$	0.00589
	$\bar{a}_{11}$	-0.00234	$\bar{b}_{11}$	0.00058
	$\bar{a}_{02}$	0.00002	$\bar{b}_{02}$	0.00004
	$\bar{a}_{30}$	-0.00011	$\bar{b}_{30}$	-0.00002
	$\bar{a}_{21}$	0.00000	$\bar{b}_{21}$	-0.00002
	$\bar{a}_{12}$	0.00000	$\bar{b}_{12}$	0.00000
	$\bar{a}_{03}$	0.00000	$\bar{b}_{03}$	0.00000
Punkt 7016	$\bar{a}_{00}$	-69.99003	$\bar{b}_{00}$	5429511.99128
	$\bar{a}_{10}$	-37576.15475	$\bar{b}_{10}$	0.47341
	$\bar{a}_{01}$	-0.00126	$\bar{b}_{01}$	-100.33634
	$\bar{a}_{20}$	-0.00121	$\bar{b}_{20}$	0.00177
	$\bar{a}_{11}$	0.00003	$\bar{b}_{11}$	0.00000
	$\bar{a}_{02}$	0.00000	$\bar{b}_{02}$	0.00000
	$\bar{a}_{30}$	0.00002	$\bar{b}_{30}$	0.00000
	$\bar{a}_{21}$	0.00000	$\bar{b}_{21}$	0.00000
	$\bar{a}_{12}$	0.00000	$\bar{b}_{12}$	0.00000
	$\bar{a}_{03}$	0.00000	$\bar{b}_{03}$	0.00000
Punkt 7220	$\bar{a}_{00}$	-76.25150	$\bar{b}_{00}$	5407273.53640
	$\bar{a}_{10}$	6272.14936	$\bar{b}_{10}$	-0.08549
	$\bar{a}_{01}$	-0.01837	$\bar{b}_{01}$	-1347.65541
	$\bar{a}_{20}$	-0.00004	$\bar{b}_{20}$	0.00005
	$\bar{a}_{11}$	-0.00007	$\bar{b}_{11}$	0.00002
	$\bar{a}_{02}$	0.00000	$\bar{b}_{02}$	0.00000
	$\bar{a}_{30}$	0.00000	$\bar{b}_{30}$	0.00000
	$\bar{a}_{21}$	0.00000	$\bar{b}_{21}$	0.00000
	$\bar{a}_{12}$	0.00000	$\bar{b}_{12}$	0.00000
	$\bar{a}_{03}$	0.00000	$\bar{b}_{03}$	0.00000
Punkt 7226	$\bar{a}_{00}$	-86.67361	$\bar{b}_{00}$	5407274.38804
	$\bar{a}_{10}$	80033.90922	$\bar{b}_{10}$	-1.23993
	$\bar{a}_{01}$	-0.00482	$\bar{b}_{01}$	-310.92454
	$\bar{a}_{20}$	-0.00682	$\bar{b}_{20}$	0.00777
	$\bar{a}_{11}$	-0.00020	$\bar{b}_{11}$	0.00005
	$\bar{a}_{02}$	0.00000	$\bar{b}_{02}$	0.00000
	$\bar{a}_{30}$	-0.00018	$\bar{b}_{30}$	-0.00003
	$\bar{a}_{21}$	0.00000	$\bar{b}_{21}$	0.00000
	$\bar{a}_{12}$	0.00000	$\bar{b}_{12}$	0.00000
	$\bar{a}_{03}$	0.00000	$\bar{b}_{03}$	0.00000

Table 2.6: Transformation from a global to a local reference system, polynomial coefficients

Punkt 6520	a_{00}	0.00000	b_{00}	5484673.72823
	a_{10}	3600.53002	b_{10}	0.04992
	a_{01}	0.05588	b_{01}	-4030.54839
	a_{20}	0.00001	b_{20}	-0.00002
	a_{11}	0.00013	b_{11}	-0.00003
	a_{02}	0.00002	b_{02}	-0.00004
	a_{30}	0.00000	b_{30}	0.00000
	a_{21}	0.00000	b_{21}	0.00000
	a_{12}	0.00000	b_{12}	0.00000
	a_{03}	0.00000	b_{03}	0.00000
Punkt 6725	a_{00}	0.00000	b_{00}	5462432.75066
	a_{10}	67263.59513	b_{10}	1.03753
	a_{01}	0.06390	b_{01}	-4142.55231
	a_{20}	0.00471	b_{20}	-0.00589
	a_{11}	0.00234	b_{11}	-0.00058
	a_{02}	-0.00002	b_{02}	-0.00004
	a_{30}	0.00011	b_{30}	0.00002
	a_{21}	0.00000	b_{21}	0.00002
	a_{12}	0.00000	b_{12}	0.00000
	a_{03}	0.00000	b_{03}	0.00000
Punkt 7016	a_{00}	0.00000	b_{00}	5429072.73102
	a_{10}	-37570.86126	b_{10}	-0.47340
	a_{01}	0.00126	b_{01}	-99.89921
	a_{20}	0.00121	b_{20}	-0.00177
	a_{11}	-0.00003	b_{11}	0.00000
	a_{02}	0.00000	b_{02}	0.00000
	a_{30}	-0.00002	b_{30}	0.00000
	a_{21}	0.00000	b_{21}	0.00000
	a_{12}	0.00000	b_{12}	0.00000
	a_{03}	0.00000	b_{03}	0.00000
Punkt 7220	a_{00}	0.00000	b_{00}	5406833.68349
	a_{10}	6271.26892	b_{10}	0.08549
	a_{01}	0.01837	b_{01}	-1347.56586
	a_{20}	0.00004	b_{20}	-0.00005
	a_{11}	0.00007	b_{11}	-0.00002
	a_{02}	0.00000	b_{02}	0.00000
	a_{30}	0.00000	b_{30}	0.00000
	a_{21}	0.00000	b_{21}	0.00000
	a_{12}	0.00000	b_{12}	0.00000
	a_{03}	0.00000	b_{03}	0.00000
Punkt 7226	a_{00}	0.00000	b_{00}	5406833.68349
	a_{10}	80022.44576	b_{10}	1.23987
	a_{01}	0.00483	b_{01}	-312.04750
	a_{20}	0.00682	b_{20}	-0.00777
	a_{11}	0.00020	b_{11}	-0.00005
	a_{02}	0.00000	b_{02}	0.00000
	a_{30}	0.00018	b_{30}	0.00003
	a_{21}	0.00000	b_{21}	0.00000
	a_{12}	0.00000	b_{12}	0.00000
	a_{03}	0.00000	b_{03}	0.00000

$\bar{x}_{10}\frac{x}{\rho}$ has been denoted by $\bar{a}_{10}$, $\bar{x}_{01}\,(\frac{y}{\rho} - y_{00})$ is called $\bar{a}_{01}$ etc.

From those tables we conclude that there are only three terms larger than a centimeter. Accordingly with such results we can reduce the computational efforts by 30 %. Indeed we need only the coefficients

$$a_{10}, a_{01}, b_{00}, b_{10}, b_{01}$$

and

$$\bar{a}_{00}, \bar{a}_{10}, \bar{a}_{01}, \bar{b}_{00}, \bar{b}_{10}, \bar{b}_{01},$$

respectively. For fast less accurate computations we can disregard the coefficients

$$a_{01} \text{ and } \bar{a}_{01}.$$

The value of such a term is smaller than 10 cm. Obviously, just for mapping purposes this accuracy is sufficient: It is an advantage when you have to compute datum transformations of conformal coordinates for *mega data sets*.

Finally we have repeated all computations by replacing the "global" reference system of type *WGS 84* by the new *World Geodetic Datum 2000* (E. GRAFAREND and A. ARDALAN 1999). *Table 2.7* reviews the best estimates of type semi-major axis A, semir-minor axis B and linear eccentricity $\varepsilon = \sqrt{A^2 - B^2}$ *both* for the tide-free geoid of reference *and* for the zero-frequency tide geoid of reference. The related data of transformation of type *UTM* (X_{84}, Y_{84}) *versus* (X_{2000}, Y_{2000}) originating from a reference system of *Bessel type* are collected in *Table 2.8* and *Table 2.9*. Indeed they document variations of the order of *a few decimeter*!

Table 2.7: World Geodetic Datum 2000 (WGS 2000) (E. GRAFAREND and A. ARDALAN 1999

	"tide-free"	
A[m]	B[m]	ε[m]
6378136.572	6356751.920	521853.58
	"zero-frequency"	
A[m]	B[m]	ε[m]
6378136.602	6356751.860	521854.674

Table 2.8: Transformation from conformal coordinates of type Gauß-Krüger (Bessel reference ellipsoid) to conformal coordinates of type Gauß-Krüger (WGS 84 and WGS 2000, "tide-free geoid")

pointnumber	X_{84}[m]	Y_{84}[m]	$X_{2000,tf}$[m]	$Y_{2000,tf}$[m]
6324	3558357.6411	5502059.3874	3558357.6374	5502059.0228
6417	3473105.6872	5488314.2616	3473105.6989	5488313.8979
6520	3503525.2908	5481082.8581	3503525.2906	5481082.4948
6725	3567188.4302	5458730.6878	3567188.4259	5458730.3259
6922	3529538.2090	5437066.5340	3529538.2071	5437066.1735
7016	3462353.8528	5429412.1301	3462353.8552	5429411.7702
7220	3506195.8794	5405925.7956	3506195.8790	5405925.4371
7226	3579947.2236	5406962.2314	3579947.2185	5406961.8728
7316	3462442.3282	5386837.1695	3462442.3306	5386836.8123
7324	3556797.3245	5387475.3087	3556797.3209	5387474.9514

Table 2.9: Transformation from conformal coordinates of type Gauß-Krüger (Bessel reference ellipsoid) to conformal coordinates of type Gauß-Krüger (WGS 84 and WGS 2000, "zero-frequency tide geoid")

pointnumber	X_{84}[m]	Y_{84}[m]	$X_{2000,zf}$[m]	$Y_{2000,zf}$[m]
6324	3558357.6411	5502059.3874	3558357.6372	5502059.0320
6417	3473105.6872	5488314.2616	3473105.6990	5488313.9072
6520	3503525.2908	5481082.8581	3503525.2906	5481082.5041
6725	3567188.4302	5458730.6878	3567188.4256	5458730.3353
6922	3529538.2090	5437066.5340	3529538.2070	5437066.1830
7016	3462353.8528	5429412.1301	3462353.8553	5429411.7796
7220	3506195.8794	5405925.7956	3506195.8790	5405925.4467
7226	3579947.2236	5406962.2314	3579947.2182	5406961.8824
7316	3462442.3282	5386837.1695	3462442.3308	5386836.8219
7324	3556797.3245	5387475.3087	3556797.3207	5387474.9610

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Abstract

A key problem of contemporary geodetic positioning is the transformation of mega data sets of conformal coordinates from a local to a global datum. Electronic mapping is another application that needs a fast solution of this problem.
Because of our case studies we can reduce the formula of Part I considerable. If we disregard the inaccuracy of the transformation parameters, the accuracy of the transformation is nearly one centimetre if we use thirty per cent of the formulae, accordingly it is a fast and efficient algorithm and a good base for a computer programme.