

# Numerical approximation of large industrial excavations

## Part one: Application of theory of finite elements

### Abstract

The first part of this paper presents the description of theory for mathematical modelling of large industrial excavations. The proposed function considers linear and non-linear approximation of a given surface.

### 1 Introduction

The considered surface can be covered by a regular or irregular net of points with known spacial coordinates. The density of these points is chosen in such a way, so that any point is calculated using equation  $z = f(x,y)$ . In consequence approximation of surface is related to Numerical Model Terrain. Various methods of model construction are given in references.

One of the first method of numerical surface modelling has been worked out by PEUCKER [7] where the surface around reference points was approximated by a plane. This method was developed further by CONNELLY [5] and RHIND [11] who introduced weighted mean values. The initial methods have been improved using skew planes or second degree planes. On the ground of this theory, the programmes were created relating the tape of the used approximation function with the number of points considered.

JUNKINS and JANCAITIS [6] have used in their programmes polynomials, which must satisfy the condition of continuity along the boundaries, determined by points, which prescribed elevations and the calculated inclination angles between points. For regular net, they use a polynomial equation of the forth degree with 15 parameters. DE MASSON and AUTUME [1] use the interpolation method derived from the method of approximation of a function with a single variables by the aid of spline curve. The approximation function has the property that integral of the square of its second derivative over the whole range of the variable constitutes a minimum. It consists of a polynomial of the third degree and it assures equality of derivatives of the first and the third degree at spline points.

BOSMAN, ECKHARD and KUBIK [4] apply the complete cubic polynomial of two variables, assuring continuity of the surface at spline points, but not providing continuity of the first derivative. BEYER [3] applies linear interpolation in a net of triangles using two nearest reference points and a suitably chosen third one. Then he performs approximations in a regular net of equilateral triangles.

BAUHUBER, ERLACHER and GUNTHER [2] present the most elaborated method in which they use a polynomial of the third degree and weighted functions for elevation interpolation.

The method suggested in this work, differs from the above mentioned elaboration's and from numerical presentation of ground surfaces included in refers [9], [12].

### 2 Theory with using of finite elements method

The linear elevation function has the following form:

$$z = L_1 Z_1 + L_2 Z_2 + L_3 Z_3 \quad (1)$$

where:  $Z_{(N)}$  – elevation at the n-th. triangle apex,

$L_{(N)}$  – functions of shape in triangle.

On the base of formula (1) we calculate elevations at the three apexes of triangle:

$$z(x_1, y_1) = L_1(x_1, y_1)Z_1 + L_2(x_1, y_1)Z_2 + L_3(x_1, y_1)Z_3$$

$$z(x_2, y_2) = L_1(x_2, y_2)Z_1 + L_2(x_2, y_2)Z_2 + L_3(x_2, y_2)Z_3$$

$$z(x_3, y_3) = L_1(x_3, y_3)Z_1 + L_2(x_3, y_3)Z_2 + L_3(x_3, y_3)Z_3$$

Definitions of functions of shape are following:

$$\text{at apex 1: } L_1(x_1, y_1) = 0 ; L_3(x_1, y_1) = 0 \text{ and } L_2(x_1, y_1) = 1 \quad (2)$$

$$\text{at apex 2: } L_1(x_2, y_2) = 0 ; L_3(x_2, y_2) = 0 \text{ and } L_2(x_2, y_2) = 1 \quad (3)$$

$$\text{at apex 3: } L_1(x_3, y_3) = 0 ; L_2(x_3, y_3) = 0 \text{ and } L_3(x_3, y_3) = 1 \quad (4)$$

Then we obtain functions of shape in form:

$$z(x_1, y_1) = Z_1 ; z(x_2, y_2) = Z_2 ; z(x_3, y_3) = Z_3$$

According to the theory of finite elements we obtain functions of elevation in triangle as follow:

$$L_1 = a_1 + b_1x + c_1y$$

$$L_2 = a_2 + b_2x + c_2y$$

$$L_3 = a_3 + b_3x + c_3y$$

Then we consider a.b. functions of elevation in apexes of triangle.

$$a_1 + b_1x_1 + c_1y_1 = 1 \quad (5)$$

$$\text{At apex 1: } a_1 + b_1x_2 + c_1y_2 = 0 \quad (5)$$

$$a_1 + b_1x_3 + c_1y_3 = 0$$

$$a_2 + b_2x_1 + c_2y_1 = 0$$

$$\text{At apex 2: } a_2 + b_2x_2 + c_2y_2 = 1 \quad (6)$$

$$a_2 + b_2x_3 + c_2y_3 = 0$$

$$a_3 + b_3x_1 + c_3y_1 = 0$$

$$\text{At apex 3: } a_3 + b_3x_2 + c_3y_2 = 0 \quad (7)$$

$$a_3 + b_3x_3 + c_3y_3 = 1$$

From the set of equations (5) we calculate coefficients;  $a_1, b_1, c_1$ , from (6);  $a_2, b_2, c_2$ , and from (7);  $a_3, b_3, c_3$ .

Determinations of set of equations (5) are:

$$\begin{aligned} W_a &= \det \begin{Bmatrix} 1 & x_1 & y_1 \\ 0 & x_2 & y_2 \\ 0 & x_3 & y_3 \end{Bmatrix} \quad W_b = \det \begin{Bmatrix} 1 & 1 & y_1 \\ 1 & 0 & y_2 \\ 1 & 0 & y_3 \end{Bmatrix} \\ W_c &= \det \begin{Bmatrix} 1 & x_1 & 1 \\ 1 & x_2 & 0 \\ 1 & x_3 & 0 \end{Bmatrix} \quad (7a) \\ W &= \det \begin{Bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{Bmatrix} = 2\Delta \end{aligned}$$

Coefficients  $a_1, b_1, c_1$  we calculate as:

$$a_1 = \frac{W_a}{W} ; \quad b_1 = \frac{W_b}{W} ; \quad c_1 = \frac{W_c}{W} \quad (8)$$

Similarly coefficients:  $a_2, b_2, c_2$  and  $a_3, b_3, c_3$  will be calculated.

Calculated coefficients are function of co-ordinates in apexes of triangle.

Then:

$$a_1 = a_1(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$b_1 = b_1(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$a_2 = a_2(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$b_2 = b_2(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$a_3 = a_3(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$b_3 = b_3(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$c_1 = c_1(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$c_2 = c_2(x_1, y_1, x_2, y_2, x_3, y_3)$$

$$c_3 = c_3(x_1, y_1, x_2, y_2, x_3, y_3)$$

In order to simplify the following calculations we assume functions of shape in form:

$$L_1 = (a_1 + b_1x + c_1y) / 2\Delta \quad (9)$$

$$L_2 = (a_2 + b_2x + c_2y) / 2\Delta \quad (10)$$

$$L_3 = (a_3 + b_3x + c_3y) / 2\Delta \quad (11)$$

where:  $2\Delta$  – double surface area of given triangle.

The  $a, b, c$  – coefficients are multiplied by  $2\Delta$  with respect to formula (8).

The simplest function [14], which assures simple interpretation of its parameters and effectiveness of numerical calculation is:

$$z = L_1Z_1 + L_2Z_2 + L_3Z_3 + P_1L_1(L_2 + L_3) + P_2L_2(L_1 + L_3) + P_3L_3(L_1 + L_2) \quad (12)$$

where:

$Z_1, Z_2, Z_3$  – constants,

$P_1, P_2, P_3$  – parameters.

Adopting this function makes it unnecessary to solve equation sets while we get the possibility to control the derivative at the triangle boundary by varying a single parameter. By comparing the directional derivatives of functions of elevation for two adjacent triangles we obtain parameter values ensuring smooth transition of surface curvatures.

We calculate the partial derivative of the elevation function (12):

$$\begin{aligned} \frac{\partial z}{\partial x} &= \frac{\partial L_1}{\partial x} Z_1 + \frac{\partial L_2}{\partial x} Z_2 + \frac{\partial L_3}{\partial x} Z_3 + \frac{P_1 \partial L_1}{\partial x} (L_2 + L_3) + \\ &+ P_1 L_1 (\frac{\partial L_2}{\partial x} + \frac{\partial L_3}{\partial x}) + P_2 \frac{\partial L_2}{\partial x} (L_1 + L_3) + \\ &+ P_2 L_2 (\frac{\partial L_1}{\partial x} + \frac{\partial L_3}{\partial x}) + P_3 \frac{\partial L_3}{\partial x} (L_1 + L_2) + \\ &+ P_3 L_3 (\frac{\partial L_1}{\partial x} + \frac{\partial L_2}{\partial x}) \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{\partial z}{\partial y} &= \frac{\partial L_1}{\partial y} Z_1 + \frac{\partial L_2}{\partial y} Z_2 + \frac{\partial L_3}{\partial y} Z_3 + \frac{P_1 \partial L_1}{\partial y} (L_2 + L_3) + \\ &+ P_1 L_1 (\frac{\partial L_2}{\partial y} + \frac{\partial L_3}{\partial y}) + P_2 \frac{\partial L_2}{\partial y} (L_1 + L_3) + \\ &+ P_2 L_2 (\frac{\partial L_1}{\partial y} + \frac{\partial L_3}{\partial y}) + P_3 \frac{\partial L_3}{\partial y} (L_1 + L_2) + \\ &+ P_3 L_3 (\frac{\partial L_1}{\partial y} + \frac{\partial L_2}{\partial y}) \end{aligned} \quad (14)$$

The derivatives of equation (9) have the following form:

$$\begin{aligned} \frac{\partial L_1}{\partial x} &= (\frac{\partial a_1}{\partial x} + \frac{\partial b_1x}{\partial x} + \frac{\partial c_1y}{\partial x}) + \frac{1}{2\Delta} ; \quad \frac{\partial L_1}{\partial y} = \\ &= (\frac{\partial a_1}{\partial y} + \frac{\partial b_1x}{\partial y} + \frac{\partial c_1y}{\partial y}) + \frac{1}{2\Delta} \end{aligned} \quad (14a)$$

We known that  $2\Delta$  is a parameter not subjected to differentiation, Y is not a function of X and the derivative is equal to zero.

Thus:

$$\frac{\partial L_1}{\partial x} = \frac{b_1}{2\Delta} \quad \frac{\partial L_2}{\partial y} = \frac{c_1}{2\Delta} \quad (15)$$

Similarly we shall calculate partial derivatives for the function of shape  $L_2$  and  $L_3$ . Their values are:

$$\frac{\partial L_2}{\partial x} = \frac{b_2}{2\Delta} \quad \frac{\partial L_2}{\partial y} = \frac{c_2}{2\Delta} \quad (16)$$

$$\frac{\partial L_3}{\partial x} = \frac{b_3}{2\Delta} \quad \frac{\partial L_3}{\partial y} = \frac{c_3}{2\Delta} \quad (17)$$

Substituting the calculated partial derivatives into formulas (13) and (14) yields:

$$\begin{aligned} \frac{\partial z}{\partial y} &= Z_1 \frac{b_1}{2\Delta} + Z_2 \frac{b_2}{2\Delta} + Z_3 \frac{b_3}{2\Delta} + P_1 L_1 \left( \frac{b_2}{2\Delta} + \frac{b_3}{2\Delta} \right) + \\ &+ P_2 L_2 \left( \frac{b_1}{2\Delta} + \frac{b_3}{2\Delta} \right) + P_1 L_2 \frac{b_1}{2\Delta} + P_1 L_3 \frac{b_1}{2\Delta} + P_2 L_1 \frac{b_2}{2\Delta} + \\ &+ P_2 L_3 \frac{b_2}{2\Delta} + P_3 L_1 \frac{b_3}{2\Delta} + P_3 L_2 \frac{b_3}{2\Delta} + P_3 L_3 \left( \frac{b_1}{2\Delta} + \frac{b_2}{2\Delta} \right) \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial z}{\partial y} &= Z_1 \frac{c_1}{2\Delta} + Z_2 \frac{c_2}{2\Delta} + Z_3 \frac{c_3}{2\Delta} + P_1 L_1 \left( \frac{c_2}{2\Delta} + \frac{c_3}{2\Delta} \right) + \\ &+ P_2 L_2 \left( \frac{c_1}{2\Delta} + \frac{c_3}{2\Delta} \right) + P_1 L_2 \frac{c_1}{2\Delta} + P_1 L_3 \frac{c_1}{2\Delta} + \\ &+ P_2 L_1 \frac{c_2}{2\Delta} + P_2 L_3 \frac{c_2}{2\Delta} + P_3 L_1 \frac{c_3}{2\Delta} + P_3 L_2 \frac{c_3}{2\Delta} + \\ &+ P_3 L_3 \left( \frac{c_1}{2\Delta} + \frac{c_2}{2\Delta} \right) \end{aligned} \quad (19)$$

We require a smooth transition of surface curvatures at the midpoint of side being common to adjacent triangles. Thus we shall calculate derivative at the triangle side midpoints. At the midpoint of side opposite to first point of triangle functions of shape assume the following values:

$$\text{for side 1: } L_1 = 0 \quad ; \quad L_2 = \frac{1}{2} \quad ; \quad L_3 = \frac{1}{2} \quad (19a)$$

The above values we enter into formulas (18) and (19):

$$\begin{aligned} \frac{\partial z}{\partial x} \Big|_{\substack{\text{for} \\ \text{side:1} \\ \text{midpo int}}} &= \frac{Z_1 b_1 + Z_2 b_2 + Z_3 b_3}{2\Delta} + P_1 \frac{b_1}{2\Delta} + \\ &+ P_2 \frac{b_1 + b_2 + b_3}{4\Delta} + P_3 \frac{b_1 + b_2 + b_3}{4\Delta} \end{aligned}$$

$$\begin{aligned} \frac{\partial z}{\partial y} \Big|_{\substack{\text{for} \\ \text{side:1} \\ \text{midpo int}}} &= \frac{Z_1 c_1 + Z_2 c_2 + Z_3 c_3}{2\Delta} + P_1 \frac{c_1}{2\Delta} + \\ &+ P_2 \frac{c_1 + c_2 + c_3}{4\Delta} + P_3 \frac{c_1 + c_2 + c_3}{4\Delta} \end{aligned}$$

We consider midpoint of side opposite to the second point of triangle.

$$\text{For side 2: } L_1 = \frac{1}{2} \quad ; \quad L_2 = 0 \quad ; \quad L_3 = \frac{1}{2}$$

Similarly we calculate the  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  derivative.

$$\begin{aligned} \frac{\partial z}{\partial x} \Big|_{\substack{\text{for} \\ \text{side:2} \\ \text{midpo int}}} &= \frac{Z_1 b_1 + Z_2 b_2 + Z_3 b_3}{2\Delta} + \\ &+ P_1 \frac{b_1 + b_2 + b_3}{4\Delta} + P_2 \frac{b_2}{2\Delta} + \\ &+ P_3 \frac{b_1 + b_2 + b_3}{4\Delta} \end{aligned}$$

$$\begin{aligned} \frac{\partial z}{\partial y} \Big|_{\substack{\text{for} \\ \text{side:2} \\ \text{midpo int}}} &= \frac{Z_1 c_1 + Z_2 c_2 + Z_3 c_3}{2\Delta} + \\ &+ P_1 \frac{c_1 + c_2 + c_3}{4\Delta} + P_2 \frac{c_2}{2\Delta} + \\ &+ P_3 \frac{c_1 + c_2 + c_3}{4\Delta} \end{aligned}$$

Now we consider midpoint of side opposite to the third point of triangle.

$$\text{For side 3: } L_1 = \frac{1}{2} \quad ; \quad L_2 = \frac{1}{2} \quad ; \quad L_3 = 0$$

Similarly we calculate the  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  derivative.

$$\begin{aligned} \frac{\partial z}{\partial x} \Big|_{\substack{\text{for} \\ \text{side:3} \\ \text{midpo int}}} &= \frac{Z_1 b_1 + Z_2 b_2 + Z_3 b_3}{2\Delta} + \\ &+ P_1 \frac{b_1 + b_2 + b_3}{4\Delta} + \\ &+ P_2 \frac{b_1 + b_2 + b_3}{4\Delta} + P_3 \frac{b_3}{2\Delta} \end{aligned}$$

$$\begin{aligned} \frac{\partial z}{\partial y} \Big|_{\substack{\text{for} \\ \text{side:3} \\ \text{midpo int}}} &= \frac{Z_1 b_1 + Z_2 b_2 + Z_3 b_3}{2\Delta} + \\ &+ P_1 \frac{c_1 + c_2 + c_3}{4\Delta} + \\ &+ P_2 \frac{c_1 + c_2 + c_3}{4\Delta} + P_3 \frac{c_3}{2\Delta} \end{aligned}$$

It has been assumed that side number 1 lies opposite to the apex number 1 in the analysed triangle and so on.

$$\text{We assume: } b_1 = y_2 - y_3 \quad ; \quad b_2 = y_3 - y_1 \quad ; \quad b_3 = y_1 - y_2$$

$$\text{Then there is always: } b_1 + b_2 + b_3 = 0$$

Thus the final forms of derivatives at the given sides midpoint are:

$$\frac{\partial z}{\partial x} \Big|_{\substack{\text{for} \\ \text{side:}i \\ \text{midpo int}}} = P_i \frac{b_i}{2\Delta} + \frac{Z_1 b_1 + Z_2 b_2 + Z_3 b_3}{2\Delta} \quad (20)$$

$$\frac{\partial z}{\partial y} \Big|_{\substack{\text{for} \\ \text{side:}i \\ \text{midpo int}}} = P_i \frac{b_i}{2\Delta} + \frac{Z_1 c_1 + Z_2 c_2 + Z_3 c_3}{2\Delta} \quad (21)$$

The directional derivative of the function of two variables is expressed by the following formulas:

$$Z' \Big|_{\substack{\text{in} \\ \text{direction}V}} = \frac{\partial z}{\partial x} V_x + \frac{\partial z}{\partial y} V_y$$

where:  $V_x$  – x co-ordinate of vector V,  
 $V_y$  – y co-ordinate of vector V.

We determine unit vector V assuming that it is perpendicular to the given side of triangle.

The equation of the straight line passing through two apexes of the triangle is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

After transformations we shall obtain the general equation of the straight line:

$$A_x + B_y + C = 0$$

where:  $A = y_2 - y_1$

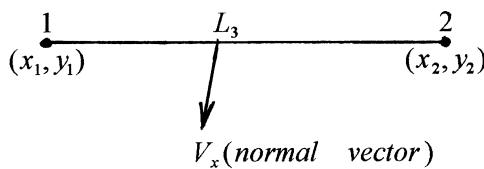
$$B = x_1 - x_2$$

$$C = y_1(x_2 - x_1) - x_1(y_2 - y_1)$$

Co-ordinates of vector normal to given straight line are:

$$V_x = A \quad \text{and} \quad V_y = B$$

That is:  $V_x = y_2 - y_1 \quad \text{and} \quad V_y = x_1 - x_2$



Having formula for directional derivatives of function of elevation for two adjacent triangles and at midpoint of common side, we can compare them by the equation:

$$\frac{\partial z^m}{\partial x} V_x + \frac{\partial z^m}{\partial y} V_y = \frac{\partial z^n}{\partial x} V_x + \frac{\partial z^n}{\partial y} V_y$$

where: m – number of considered triangle,  
n – number of adjacent triangle.

Substituting formulas (20) and (21) into the above equation yields:

$$P_i^m \left( \frac{b_i^m}{\Delta^m} V_x + \frac{c_i^m}{\Delta^m} V_y \right) + \frac{1}{\Delta^m} \sum_{i=1}^{i=3} z_i^m (b_i^m V_x + c_i^m V_y) =$$

$$= P_j^n \left( \frac{b_j^n}{\Delta^n} V_x + \frac{c_j^n}{\Delta^n} V_y \right) + \frac{1}{\Delta^n} \sum_{j=1}^{j=3} z_j^n (b_j^n V_x + c_j^n V_y)$$

where: i – number of side of the considered triangle,  
j – number of adjacent triangle.

After substitution:

$$T_i^m = \frac{b_i^m}{\Delta^m} V_x + \frac{c_i^m}{\Delta^m} V_y \quad (22)$$

$$T_j^n = \frac{b_j^n}{\Delta^n} V_x + \frac{c_j^n}{\Delta^n} V_y \quad (23)$$

$$S_i^m = \frac{1}{\Delta^m} \sum_{i=1}^{i=3} z_i^m (b_i^m V_x + c_i^m V_y) \quad (24)$$

$$S_j^n = \frac{1}{\Delta^n} \sum_{j=1}^{j=3} z_j^n (b_j^n V_x + c_j^n V_y) \quad (25)$$

Applying the above notation yields:

$$P_i^m T_i^m + S_i^m = P_j^n T_j^n + S_j^n$$

Thus:

$$P_j^n = P_i^m \frac{T_i^m}{T_j^n} + \frac{S_i^m - S_j^n}{T_j^n} \quad (26)$$

Then we must establish a second equation and calculate the unknown parameters. We shall apply condition of the minimum of sum of parameters squared:

$$(P_i^m)^2 + (P_j^n)^2 = \text{MINIMUM}$$

In this way, by balancing the parameter distribution we prevent incorrect function of shape. Substituting formula for  $P_j^n$  into the above formula yields:

$$(P_i^m)^2 + (P_i^m \frac{T_i^m}{T_j^n})^2 + 2(P_i^m \frac{T_i^m (S_i^m - S_j^n)}{(T_j^n)^2}) + (\frac{S_i^m - S_j^n}{T_j^n})^2 = \\ = \text{MINIMUM}$$

In order to satisfy this condition that derivative of the equations left hand side must be equal to zero. After calculations we shall obtain:

$$P_i^m = \frac{T_i^m (S_j^n - S_i^m)}{(T_j^n)^2 + (T_i^m)^2} \quad (27)$$

The second parameter  $P_j^n$  we shall calculate from formula (26):

$$P_j^n = \frac{S_j^n - S_i^m}{T_j^n} \left( \frac{(T_i^m)^2}{(T_j^n)^2 + (T_i^m)^2} - 1 \right) \quad (28)$$

### 3 Conclusions

The paper presents a new suggestion of the modelling of the large industrial excavation surfaces by the aid of the shape function. The numerical approximation of considered surfaces requires solving the following problems:

- zonation of a given surface,
- automatic connection of the measuring points with irregular dispersion,
- automatic generation of irregular triangles net.

The above problems has been presented by the author in paper [10]. The method of mathematical approximation of mentioned surfaces can be applied for calculations of: profile points heights, embankments or fragments of rivers capacity by the aid of integration of the shape function, volumes of silt on the rivers bottom, and in the particular cases for precisely mathematical modelling of irregular surfaces of embankments.

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## Deutsche Gaswirtschaft geht online

- „BGW-Gasnetzkarte“ mit SMALLWORLD im Internet
- Transportentgelte individuell ermitteln
- GIS-Applikation, ASP und E-Commerce als integrierte Lösung

Während die Liberalisierung des Strommarktes die geografischen Netzlängen und Gebiete nur zu einem geringen Teil in die Ermittlung der Transportentgelte einbezieht, so sagt die Anfang Juli 2000 verabschiedete „Verbändevereinbarung zum Netzzugang bei Erdgas“: „Der BGW verpflichtet sich, bis zum 10. August 2000 eine Netzkarte zu veröffentlichen, die nachfolgend genannte Angaben enthalten wird: Netzkarte des Erdgasleitungssystems... mit Bezeichnung der Netzbetreiber, Nenndurchmesser und Nenndruck sowie Kennzeichnung der Verbindungsstellen.“

Die Vereinbarung bezieht sich auf ca. 70 000 km Erdgasleitungsnetz in Deutschland und erfordert eine Netzlängenberechnung auf geografischer Basis, wie sie von geographischen Informationssystemen geleistet werden kann. Die Gasnetzkarte des Bundesverbandes der deutschen Gas- und Wasserwirtschaft e.V. („BGW-Gasnetzkarte“) stellt dabei das deutsche Gas-Transportnetz auf der Basis von 1994 im

Maßstab 1:200 000 zur Verfügung.

Der BGW beauftragte SMALLWORLD, diese Karte in digitaler Form, versehen mit Navigationshilfen, Identifikations-, Auskunfts möglichkeiten und Netzroutingfunktionalitäten, über das Internet bereitzustellen.

SMALLWORLD stellt diese Applikation zum Beginn der Gültigkeit der „Verbände vereinbarung Gas“ seit dem 29. August 2000 unter [www.gasnetzkarte.de](http://www.gasnetzkarte.de) online bereit. Die Anwendung basiert auf der in Gemeinschaftsentwicklung mit sechs Transportleitungsunternehmen entstandenen Fachschale Ferngas sowie dem neuen Internet-Application-Server (SIAS) von SMALLWORLD, der hier erstmals eingesetzt wird.

Die für die Abbildung der Inhalte der „BGW-Karte“ geschaffenen Applikationen wurden speziell an die im Internet angebotenen Informationsinhalte angepasst. Die Applikation bildet sowohl die topographischen als auch leitungstechnischen Inhalte der Karte ab, ergänzt durch die digitalen Gemeindegrenzen der Bundesrepublik. Auf diese Weise wird ein ortsbegrener Zugriff möglich, der als Navigationskriterium und zur Festlegung von möglichen Transportwegen genutzt werden kann. Die in

der „BGW-Karte“ abgebildeten Angaben zu Leitungseigentümer, Druckstufe und Nennweite können durch Anklicken in der Karte identifiziert werden. Die themengesteuerte Ein- und Ausblendung von Karteninhalten ist ebenso möglich, wie die freie Wahl des Bildschirmausschnittes durch Zoom-Möglichkeiten.

Die Bestimmung der Transportentgelte setzt die Ermittlung des jeweils kürzesten Weges auf dem Leitungsnetwork voraus, welcher durch Wahl des Anfangs- und Zielpunktes analysiert werden kann. Ergebnis einer jeden Anfrage sind Angaben zu den Leitungsabschnitten, die dem Anwender zum Abruf als Liste bereitgestellt werden. Gleichzeitig wird der entsprechende Kartenausschnitt bereitgestellt.

Die Vergütung der über das Internet verfügbaren Funktionalitäten erfolgt pro Anfrage (transaktionsorientiert) über die Einbindung einer E-Commerce-Software. Die einzelnen Anfragen lassen sich wie in einem elektronischen Kaufhaus in einen Warenkorb speichern, der dann die Endsumme aller Anfragen pro Zugriff bildet. Für diese spezielle Funktion wurde Software von Inter shop eingesetzt.

Nächste Schritte im Projekt sind die Aktualisierung der

Gasnetzkarte auf den aktuellen Stand des Jahres 2000. Aus der Sicht von SMALLWORLD kann die Möglichkeit des Bezuges der Daten zur eigenen Verarbeitung in entsprechenden Analysesystemen eine weitere Anforderung sein, um dann unternehmensspezifische Handelssysteme in Kopplung mit Leittechnik- und Billingsystemen aufzubauen. Gleichfalls könnten bei Kenntnis der Leistungspreise der Transportgesellschaften die Durchleitungsentgelte als Teil des Transportentgeltes kalkuliert werden. Die zukünftige Weiterentwicklung wird sich an den jeweils aktuellen Inhalten der Verbändevereinbarung Gas ausrichten.

SMALLWORLD möchte auf diese Weise seine führende Rolle als Technologielieferant in der deutschen Gas-Versorgungsindustrie unterstreichen und die Kompetenz zur Unterstützung der aktuellen Fragestellungen und Unternehmensprozesse verdeutlichen.

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