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Geoid surface approximation by using Adaptive Network based Fuzzy Inference Systems

Die Fuzzy-Theorie und neue Bewertungen von Unsicherheiten erlauben die präzise Modellierung geodätischer Daten. In der vorliegenden Arbeit wurden unter Verwendung des adaptiven Network Fuzzy Inference System (ANFIS) Geoidhöhen in den Regionen Baden-Württemberg (Deutschland) und in Izmir (Türkei) mit Erfolg bestimmt und die Fuzzy Modelle mit vielen Testpunkten überprüft. Damit wurde gezeigt, dass das hier verwendete Verfahren bessere Ergebnisse als die herkömmlichen Verfahren liefert und für die Lösung weiterer geodätischer Probleme in Betracht gezogen werden sollte.

1 Introduction

The shape of the earth and its physical structure have been described by GAUSS [4] and later named as geoid by LISTING [10].
The early definition and reasoning of the geoid and its physical aspects by GAUSS are still valid. Moreover, the recent technological developments have enabled geodesists to establish large geodetic networks consist of points with known coordinates in the same coordinate system as GAUSS mentioned in early 1800s.
The studies on the determination of the best fitting Earth-ellipsoid have been carried on since the early times and as the geodetic datum, different ellipsoids with different parameters (semi-major axis, flattening, etc.) have been calculated. However, one of them is particularly of interest – World Geodetic System 1984 (WGS-84) that is the coordinate system of the Global Positioning System (GPS) based on observations to artificial satellites.
Nowadays, gravimetric method is the most commonly used technique for the precise determination of the geoid. The precondition for its use is the presence of high-resolution gravity data set. With the lack of gravity data the geoid could be determined by means of various geometric methods: astro-geodetic method or geoid heights from GPS in conjunction with spirit levelling [9].
The need for refined models of the geoid has been driven principally by the demands of users of the GPS, who must transform GPS-derived ellipsoidal heights to orthometric heights, e.g. [2] in order to make them compatible with the existing orthometric heights on the local vertical datum. The orthometric heights are determined by spirit levelling. The ellipsoidal height is reckoned, along the ellipsoidal normal, from the surface of any reference ellipsoid to the point of interest. The orthometric height is reckoned, along the curved plumbline, from the surface of the geoid to the point of interest. The geoid height or geoid-ellipsoid separation is reckoned, along the ellipsoidal normal, from the surface of any reference ellipsoid to the geoid. The transformation of ellipsoidal heights to orthometric heights therefore requires that the geoid height must refer to the same reference ellipsoid [3].
Regarding the above definitions, the geoid height at each point is achieved by

$$\mathbf{N} \approx \mathbf{h} - \mathbf{H} \tag{1}$$

where N is the geoid height, h is the ellipsoidal height and H is the orthometric height. The geometrical relation between N, h and H is shown in Fig. 1.
The approximate equality in Eq. (1) results from neglecting the departure of the plumbline from the ellipsoidal normal, which is termed the deflection of the vertical. It is acknowledged that there is also torsion in the plumbline, but the deflection of the vertical is usually the dominant effect of the approximation in Eq. (1). The approximation error can be estimated by multiplying the orthometric height by the cosine of the deflection of the vertical at the point of interest. However, this approximation error is in maximum 1 – 2 mm level and is considerably smaller than the accuracy with which GPS-derived ellipsoidal and orthometric heights can currently be determined. Therefore, the approximation in Eq. (1) remains valid for the transformation of heights.

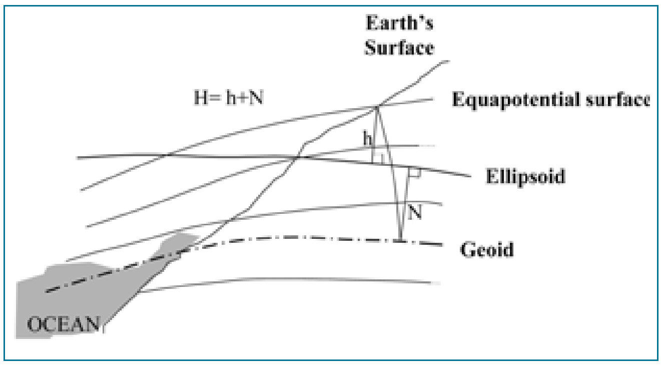

Fig. 1: The geometrical relationship between orthometric and ellipsoidal heights.

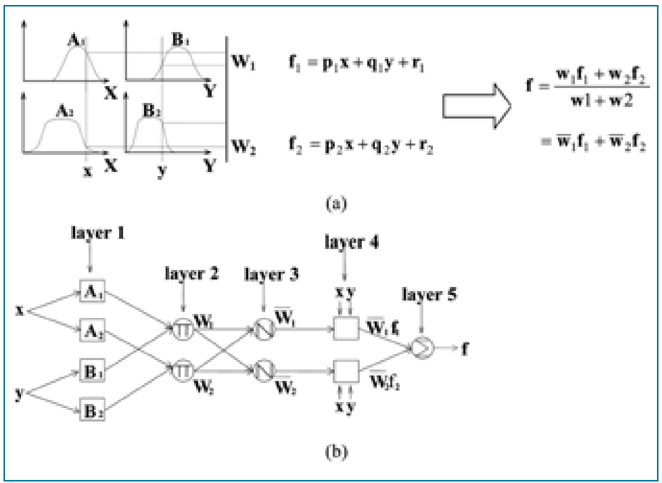

Fig. 2(a), (b): (a) Type-3 fuzzy reasoning, (b) Equivalent ANFIS (Type-3 ANFIS)

In the following sections, geoid at Baden-Württemberg (Germany) and Izmir (Turkey) have been modelled by means of adaptive neuro-fuzzy inference systems and the results have been interpreted from the physical point of view.

2 Adaptive Network based Fuzzy Inference Systems

Adaptive Network based Fuzzy Inference Systems (ANFIS) are feed-forward adaptive networks which are functionally equivalent to fuzzy inference systems. The basic idea of ANFIS can be described as follows: A fuzzy inference system is typically designed by defining linguistic input and output variables as well as an inference rule base. However, the resulting system is just an initial guess for an adequate model. Hence, its premise and consequent parameters have to be tuned based on the given data in order to optimise the system performance. In ANFIS this step is based on a supervised learning algorithm.

All types of fuzzy inference systems shown in Fig. 2 can be subjected to such a procedure. However, the complexity of the problem depends on the type of reasoning in the consequent part even if the results of all three types would not change significantly for the same data set. Therefore, in this section, Type-3 ANFIS is explained which is least complex and hence used for the prediction of the geoid heights.

For simplicity, assume that the fuzzy inference system under consideration has two inputs x and y and one output f. Additionally, suppose that the rule base contains two fuzzy *if-then* rules of Takagi and Sugeno's type [13] as

Rule 1: If x is A_1 and y is B_1; then $f_1 = p_1x + q_1y + r_1$.

Rule 2: If x is A_2 and y is B_2; then $f_2 = p_2x + q_2y + r_2$.

The associate Type-3 fuzzy reasoning is illustrated in Fig. 2(a), and the corresponding equivalent ANFIS architecture (Type-3 ANFIS) is shown in Fig. 2(b).

Note that the node functions in the same layer are of the same function family (all circles without parameters or square nodes with parameters).

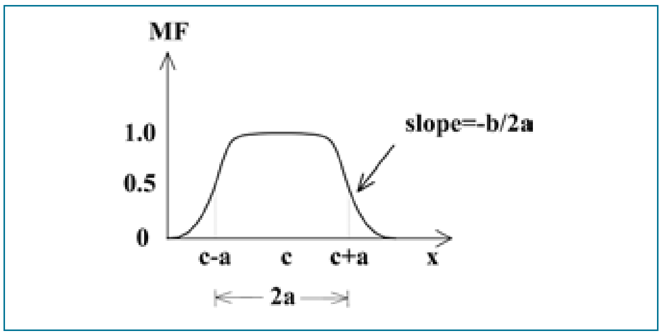

Fig. 3: Meanings of the parameters in the generalised bell membership function

The functions of each layer can be described as below.

Layer 1: Every node i in this layer is a square node with a node function

$$O_i^1 = \mu_{A_i}(x); \tag{2}$$

where x is the input to node i, and A_i is the linguistic label (small, medium, large, etc.) associated with this node function. In other words, O_i^1 is the membership function of A_i and it specifies the degree to which the given x satisfies the quantifier A_i. Usually, the membership function $\mu_{A_i}(x)$ is chosen to be bell-shaped with the maximum value equal to 1 and the minimum value equal to 0 such as, e.g., the generalised bell function (Fig. 3)

$$\mu_{A_i}(x) = \frac{1}{1 + \left[\left(\frac{x-c_i}{a_i}\right)^2\right]^{b_i}}, \tag{3}$$

or the Gaussian function

$$\mu_{A_i}(x) = \exp\left[-\left(\frac{x-c_i}{a_i}\right)^2\right], \tag{4}$$

where $\{a_i,\ b_i,\ c_i\}$ (or $\{a_i,\ c_i\}$ in case of the Gaussian function) is the parameter set. As the values of these parameters change, the bell-shaped functions vary accordingly. Thus various membership functions on linguistic label A_i are defined. In fact, any continuous and piecewise differentiable functions, such as commonly used trapezoidal or triangular-shaped membership functions can also be considered as qualified candidates for node functions in this layer. Parameters in this layer are called "premise parameters".

Layer 2: Every node in this layer is a circle node, which performs a fuzzy intersection operation on the incoming signals from the first layer and sends the result to the next layer. For instance,

$$w_i = \mu_{A_i}(x) \cdot \mu_{B_i}(y) \text{ or } w_i = \min[\mu_{A_i}(x), \mu_{B_i}(y)], i = 1, 2. \tag{5}$$

The left equation shows fuzzy intersection by the algebraic product, the second one the minimum intersection as they are called. Both variants are consistent extensions of intersection in classical set theory. Please note that each node output represents the firing strength of a rule (Fig. 2).

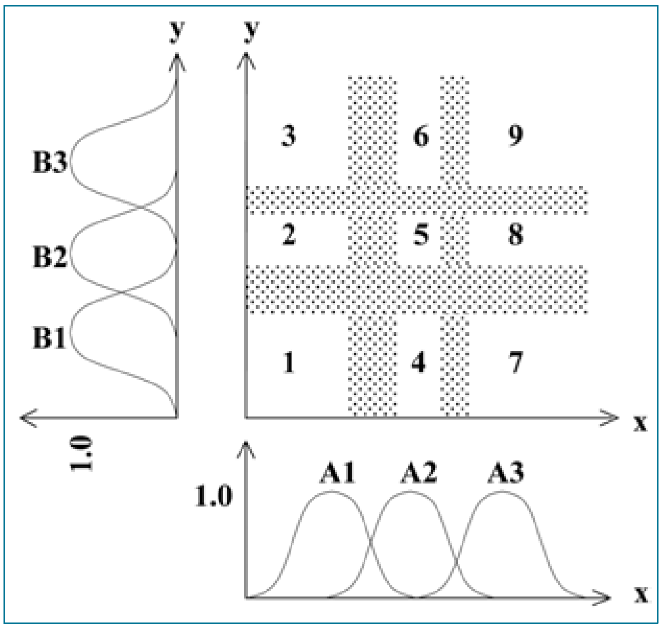

Fig. 4: Corresponding fuzzy subspaces for a two-input Type-3 ANFIS with 9 rules

Layer 3: Every node in this layer is a circle node such that the i-th node calculates the ratio of the i-th rule's firing strength to the sum of all rules' firing strengths as

$$\bar{w}_i = \frac{w_i}{w_1 + w_2}, i = 1, 2. \tag{6}$$

Outputs of this layer can be called normalised firing strengths.
Layer 4: Every node in this layer is a square node with a node function that calculates the output for corresponding rules weighted by its normalised firing strength such that

$$O_i^4 = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i), \tag{7}$$

where $\bar{w}_i$ is the output of the previous layer (layer 3), and $\{p_i,\ q_i,\ r_i\}$ is the set of parameters which are called "consequent parameters".
Layer 5: The single node in this layer is a circle node that computes the overall output by using the weighted average defuzzification method as

$$O_i^5 = overall\ output = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{8}$$

Fig. 4 shows an example of fuzzy partitioning of the input space in case of two inputs. Each of them is represented by three membership functions. So the input space is partitioned into nine fuzzy subspaces thus leading to nine fuzzy *if-then* rules in the ANFIS.

2.1 Hybrid Learning Algorithm adopted in ANFIS reasoning

The hybrid learning algorithm used in ANFIS modelling is the combination of Least-Squares estimation and gradient descent method. The advantage of this combined algorithm vs. the pure gradient descent is that the rapid convergence to the global minimum is guaranteed whereas the gradient descent is usually slow and likely to become trapped in local minima [6, 7, 14].
For simplicity, assume that the adaptive network under consideration has only one output,

$$output = F(\vec{I}, \tag{9}$$

where $\vec{I}$ is the set of input variables and S is the set of parameters. If there exists a function H such that the composite function $H \cdot F$ is linear in some of the elements of S, then these elements can be identified by the least-squares method [7]. Note that the Type-3 ANFIS satisfies this condition rather well since it contains linear parameters in the consequent part (consequent parameters).
Let S be the total parameters of the considered ANFIS model which is decomposed into two sets as

$$S = S_1 \oplus S_2, \tag{10}$$

where $\oplus$ represents the direct sum. S_1 and S_2 are the sets of premise and the consequent parameters, respectively. Recalling the sample ANFIS architecture given in Fig (2) and assuming that the values of the premise parameters are given, the overall output can be expressed as a linear combination of the consequent parameters. More precisely, the output f in Fig. (2) can be rewritten as

$$f = \frac{w_1}{w_1 + w_2} f_1 + \frac{w_2}{w_1 + w_2} f_2 =$$
$$\bar{w}_1 f_1 + \bar{w}_2 f_2 = (\bar{w}_1 x)p_1 + (\bar{w}_1 y)q_1 + (\bar{w}_1)r_1 +$$
$$(\bar{w}_2 x)p_2 + (\bar{w}_2 y)q_2 + (\bar{w}_2)r_2, \tag{11}$$

which is linear in the consequent parameters $\{p_1, q_1, r_1, p_2, q_2, r_2\}$. In this case, the formerly mentioned functions $H(\cdot)$ and $F(\cdot,\cdot)$ are the identity function and the function of the fuzzy inference system, respectively.
Given the values of the elements of S_1, one can write a system of equations regarding the number P of training data pairs in matrix form as

$$\mathbf{AX} = \mathbf{B}, \tag{12}$$

where $\mathbf{X}$ is the unknown vector whose elements are the parameters in S_2. Let $|S_2| = M$, then the dimensions of $\mathbf{A}$, $\mathbf{X}$ and $\mathbf{B}$ are $P \times M$, $M \times 1$ and $P \times 1$, respectively.
Since the number of training data is usually greater than the number of unknowns, a least- squares estimate (LSE) $\hat{\mathbf{X}}$of $\mathbf{X}$ is sought to minimise the squared error norm $\|\mathbf{AX} - \mathbf{B}\|^2$. The solution of the respective normal equations system is well known

$$\hat{\mathbf{X}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B} \tag{13}$$

[8]. $(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ s the pseudo-inverse of $\mathbf{A}$ when $\mathbf{A}^T\mathbf{A}$ is non-singular. However, this methodology is expensive in the computation of the matrix inverse and, moreover, it becomes ill defined if $\mathbf{A}^T\mathbf{A}$ is singular. Thus, a LSE based on sequential formulas such as Kalman filtering could be replaced by the direct solution given in Eq. (13). This sequential method is more efficient, in particular when the number of unknowns is small. Let a_i^T and b_i^T be the i-th row vector of the matrix A and the i-th element of vector B, respectively, then the so-called iterative solution reads as

$$\begin{aligned} X_{i+1} &= X_i + C_{i+1} a_{i+1}(b_{i+1}^T - a_{i+1}^T X_i) \\ C_{i+1} &= C_i - \frac{C_i a_{i+1} a_{i+1}^T C_i}{1 + a_{i+1}^T C_i a_{i+1}}, \quad i = 0,\ 1, \ldots, P-1 \end{aligned} \tag{14}$$

Tab. 1: Two passes in the hybrid learning procedure for ANFIS

–	FORWARD PASS	BACKWARD PASS
premise parameters	fixed	gradient descent
consequent parameters	Least-squares estimate	fixed
Signals	Node outputs	error rates

where C_i is the covariance matrix. The LSE of X is equal to X_P [5, 11, 12, 14]. The initial conditions in Eq. (14) are $X_0 = 0$ and $C_0 = \gamma I$, where γ is a non-negative large number and I is an identity matrix with dimension M × M. In case of multi-output fuzzy inference systems, Eq. (14) still runs except that b_i^T is the *i*-th row vector of matrix **B**.
The hybrid learning algorithm used in ANFIS is composed of a forward pass and a backward pass for each epoch. In the forward pass, the premise parameters are kept fixed and the functional signals coming from the input data go forward to calculate each node output until the matrices **A** and **B** in Eq. (12) are obtained. The vector of the consequent parameters S_2 is given by Eq. (14). Then the functional signals keep going forward until the error measure is calculated. In the backward pass, the error rates (the derivatives of the error measure with respect to each node output) are transferred from the output end towards the input end using chain rule. The premise parameters S_1 are then updated by the gradient method [7]. See Table 1 for a compilation.
For given fixed values of the premise parameters, the consequent parameters thus found are guaranteed to be the global optimum point in the S_2 parameter space due to the choice of the squared error measure. Accordingly, the hybrid approach is much faster than the strict gradient descent [7].

3 Numerical Examples

The determination of the geoid is in fact the interpolation of the known geoid heights at the control points that are located properly on the ground. However, the accuracy of the geoid depends on the accuracy of the input data, i.e. the accuracy and the density of the known geoid heights rather than the method used for interpolation. Up to now, different methods such as least squares collocation, multi-parameter polynomial fitting, multi-quadratic interpolation or weighted linear interpolation by different auto-covariance functions have been widely employed for this purpose. All these methods have been evaluated only from the mathematical point of view, and have often neglected the physical aspects of the geoid.
As an alternative to the classical approximation models, adaptive network based fuzzy inference systems have been used to approximate geoid heights as a function of geographic coordinates φ, λ and ellipsoidal height **h**. Therefore, the input-output pair consists of ellipsoidal geographic coordinates φ, λ and **h** as input variables, and geoid height **N** as single output variable. In this alternative approach, the selected data set has been subjected to a normalisation procedure.
Established ANFISs are trained regarding the training data set by the fore-mentioned hybrid algorithm until the average root mean square errors for both training and testing points are minimum. Optimal network configurations (number of fuzzy sets in each input variable space thus leading the number of fuzzy rules) have been obtained by trials. However, finding the optimal configuration is very less complicated and time-consuming than those in Artificial Neural Networks (ANN).

Izmir test area

Figure 5 shows the region of Izmir, covering the area of approximately $d\varphi = 0.30°$ and $d\lambda = 0.56°$. The geoid height in the area of interest vary from 37.6 to 38.7 m. The actual geoid heights are known in 310 points [1]. 75 points out of these 310 points (in Fig. 5 marked with squares) were randomly selected to form a test set. The remaining 235 points (in Fig. 5 marked with filled circles) have been used for training the ANFIS network. In addition, all 310 points were used for the 5^{th} order polynomial fitting of the geoid at the region [1]. The training procedure of ANFIS was performed using different number of fuzzy sets in each input variable which means that different number of fuzzy rules were employed. The best configuration was found to use three, five and one Gaussian type fuzzy membership functions (fuzzy sets) for φ, λ and h, respectively.
The efficiency of the ANFIS approximation was compared to the results obtained by the 5^{th} order polynomial model. The performance of the ANFIS approximation was tested at all 235 points and at 75 new points. The differences between actual geoid heights and approximated values are summarised in Table 2. It can be seen that ANFIS approximation results are considerably better than the results obtained by 5^{th} order model.
Though the point density is high in the region, the average accuracy (standard deviation) of the ellipsoidal heights after the adjustment of the network has been found to be ± 3.5 cm. The accuracy of ellipsoidal heights vary from ± 2.8 cm to ± 5.0 cm. Additionally, regarding the

Tab. 2: Comparison between ANFIS and the fifth order approximations (Izmir)

Approximation type	Min. [m]	Max. [m]	Mean [m]	St. dev. [m]	Correlation Coefficient
235 points ANFIS	– 0.111	0.098	0.000	0.030	0.98335
310 points 5^{th} order	– 0.156	0.122	0.000	0.044	0.96527
75 points ANFIS	– 0.090	0.141	0.000	0.037	0.97222

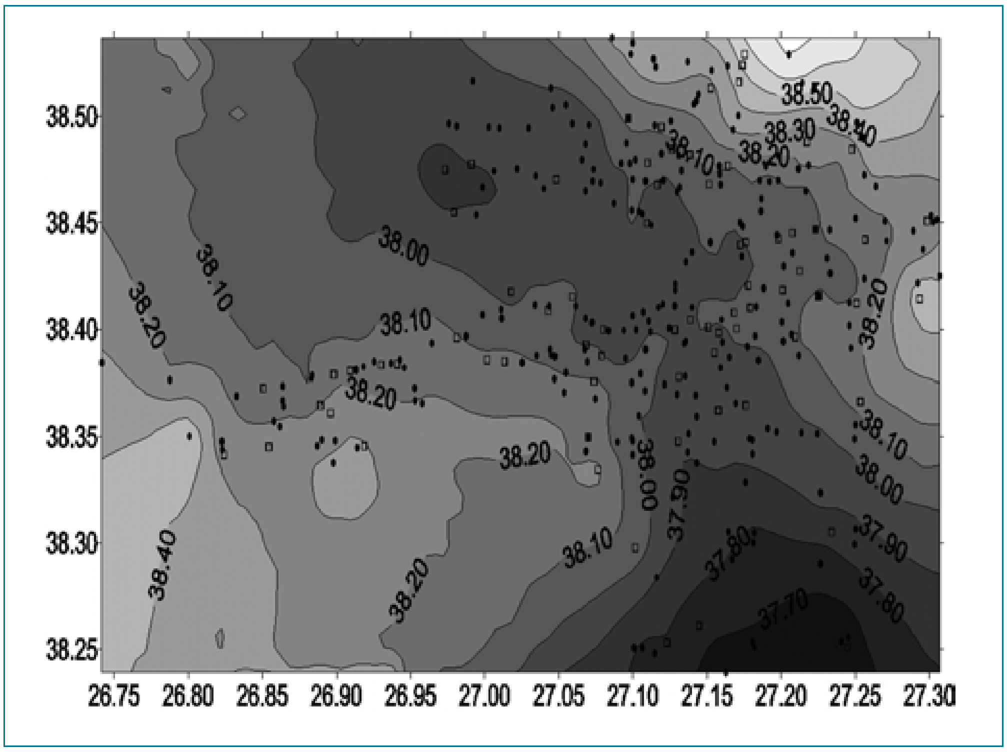

Fig. 5: Actual geoid shape in Izmir region

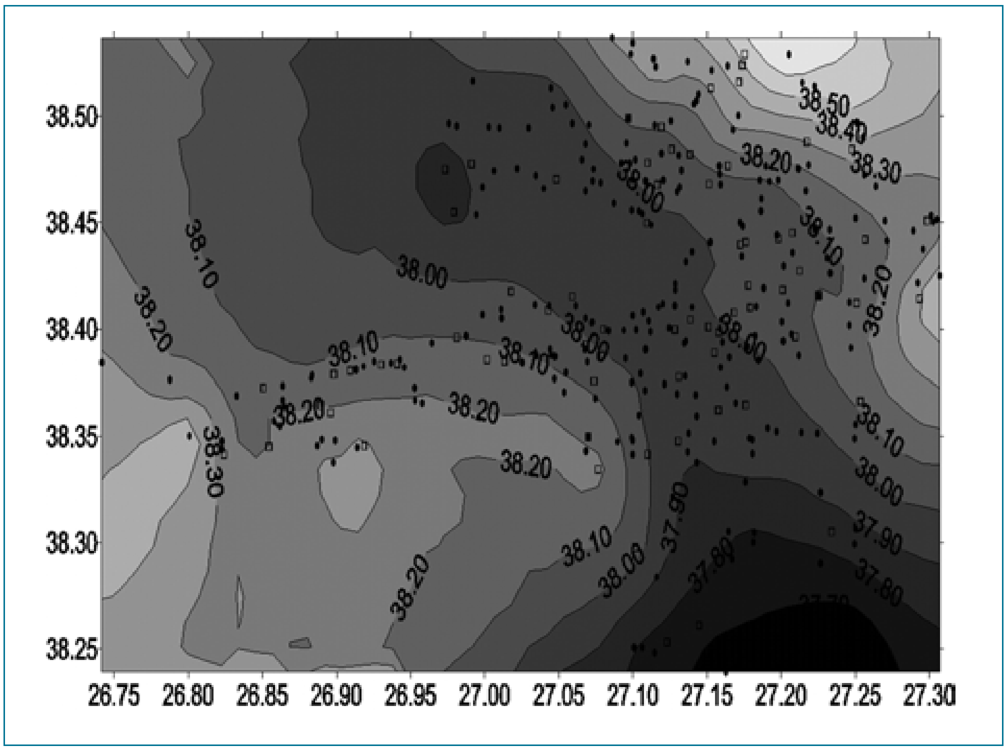

Fig. 6: ANFIS approximation of geoid in Izmir region

Fig. 5, the point distribution is not very convenient especially towards the Izmir gulf. Because of all these reasons, achieved approximation accuracy by ANFIS could be assumed to be perfect against those of classical approaches.

Baden-Württemberg test area

In this case the quality of ANFIS approximation was tested over a larger area. In order to make a comparison with a previous work done by using Artificial Neural Networks [9], from the Prof. Wenzel's web page – GPS/Levelling derived geoid heights [15], 125 points were chosen in the state of Baden-Württemberg, covering the area of approximately $\Delta\varphi = 2^\circ$ and $\Delta\lambda = 3^\circ$. Geoid heights in this area vary from 46.6 to 50.2 m (Fig. 8).

In order to follow the same conditions with [9], 99 points among 125 points were randomly selected for training the

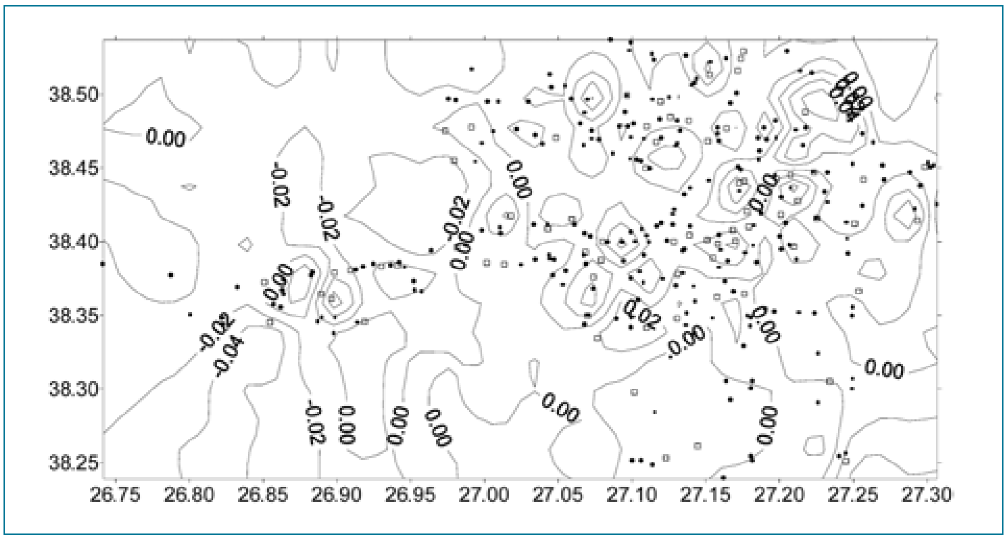

Fig. 7: Differences between geoid and ANFIS approximation of geoid in Izmir region

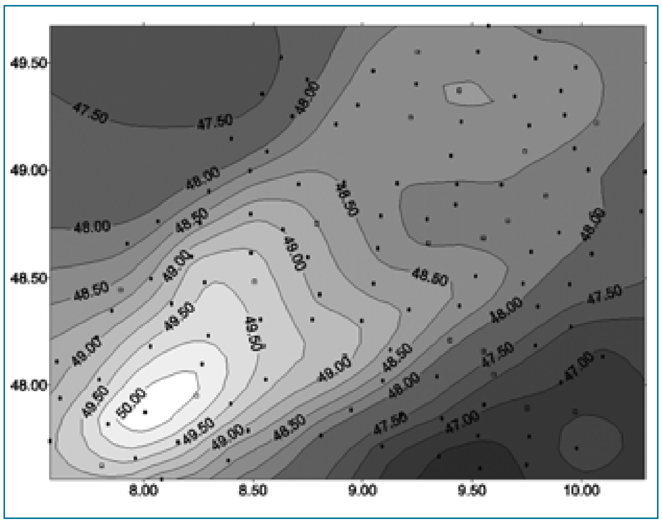

Fig. 8: Actual quasigeoid shape in Baden-Württemberg

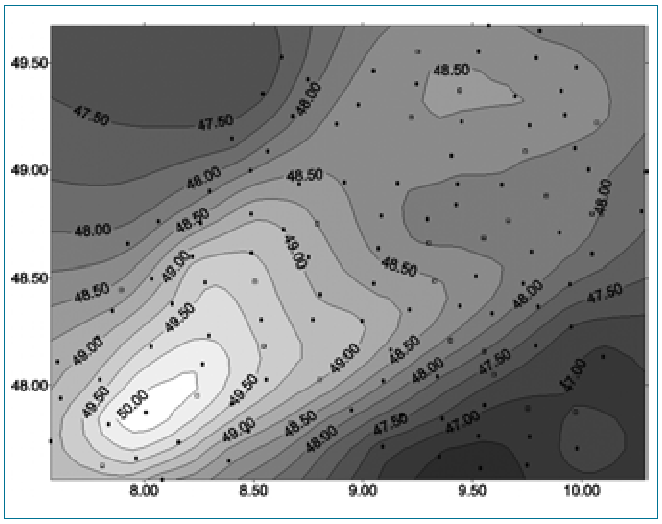

Fig. 9: ANFIS approximation of quasigeoid in Baden-Württemberg

network. The remaining 26 points has been used to form the test set. In this case, optimal ANFIS architecture was found to use four, three and one Gaussian type fuzzy membership functions (fuzzy sets) for φ, λ and h, respectively. The quasigeoid shape of the test area obtained by ANFIS approximation is shown in Fig. 9, and the differences between the actual quasigeoid shape and the quasigeoid ANFIS approximation are given in Fig. 10.

Some measures of the achieved ANFIS approximation quality at the area were given together with the results of [9] in the same sense in Table 3.

From Table 3, it is easy to conclude that the quality measures obtained for both training points and the new points by ANFIS is twice superior to those obtained by ANFIS. Looking at the Table 3, only the maximum error value among 26 new points of ANFIS approximation is very close to that of ANN approximation. However, when the standard deviations are compared, one can say that the amount of high fitting errors in ANFIS approach is

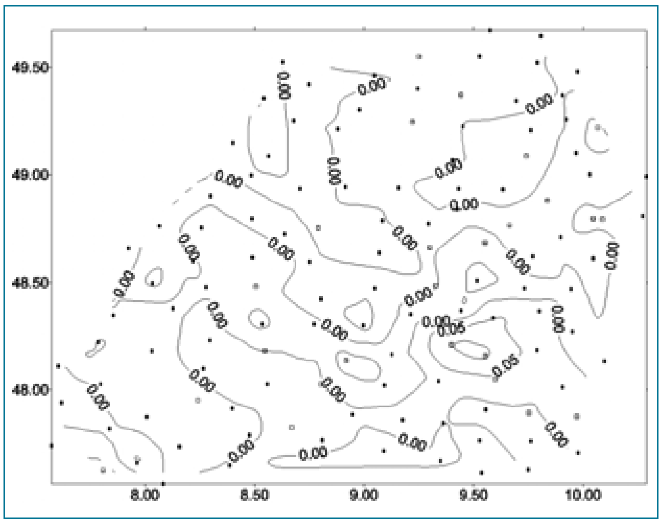

Fig. 10: Differences between quasigeoid and ANFIS approximation of quasigeoid in Baden-Württemberg

Tab. 3: Accuracy of ANFIS approximation (this study) and ANN approximation by [9]

Approximation type	Min. [m]	Max. [m]	Mean [m]	St. dev. [m]	Correlation Coefficient
99 points ANFIS	– 0.070	0.067	0.000	0.029	0.99933
125 points ANFIS	– 0.073	0.113	0.000	0.033	0.99907
125 points ANN	– 0.189	0.185	0.002	0.076	0.99517
26 points ANFIS	– 0.073	0.112	0.001	0.047	0.99827
26 points ANN	– 0.126	0.113	0.001	0.067	0.99633

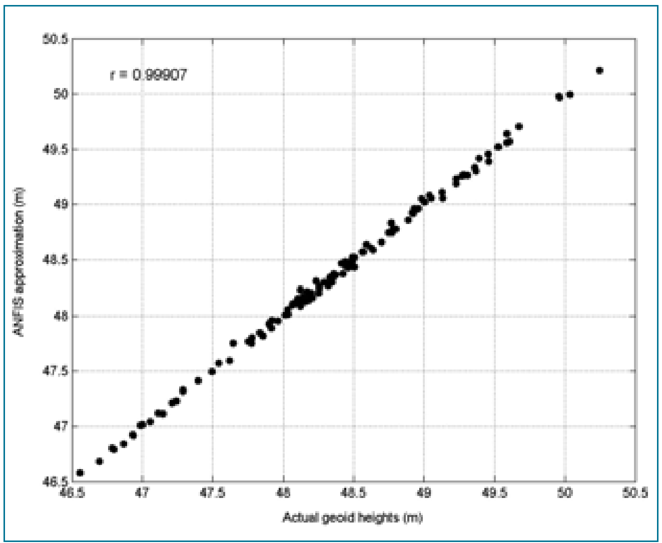

Fig. 11: Comparison of actual quasigeoid heights and ANFIS quasigeoid approximation in Baden-Württemberg

considerably less than that of ANN approach. Achieved accuracy also depends on the fine distribution of the points in the region of Baden-Württemberg, which was not met in region of Izmir.

The correlation coefficient value computed for the actual geoid heights and those obtained by ANFIS approximation is again quite high and higher than those of ANN. Correlation coefficient between ANFIS and actual geoid heights computed with 125 points is shown in Fig. 11.

Conclusion

As formerly pointed out, our investigation focuses on the application of Adaptive Network based Fuzzy Inference System (ANFIS) on geoid height determination. To make an objective conclusion, the outputs have been compared with the results of different approximation methods, i.e. high order polynomial fitting and artificial neural networks. The results of ANFIS approximation have been found to be superior to all other methods.

While using ANFIS, one has to care some aspects such that the number of parameters (including both premise and consequent) in ANFIS have to be less than the number of training data pairs. This is for avoiding the overfitting phenomenon, which does not allow generalisation of the established fuzzy inference system.

No matter which method is used, the efficiency of the approximation is limited at least with the accuracy of the ellipsoidal heights obtained from the adjustment of the network.

This means that it is not possible to achieve an approximation accuracy better than the accuracy of the ellipsoidal heights. Considering this limitation, the standard deviation obtained by ANFIS (for Izmir example) is very close to the accuracy of adjusted ellipsoidal heights. In addition, point distribution is also an important factor for the approximation quality.

Though it is not mentioned in the text, the convergence of ANFIS is considerably faster than that of ANN.

The data in this study consist of ellipsoidal heights obtained by GPS technique and orthometric heights obtained by levelling. However, in case gravity data is available, the fuzzy model could be extended by the inclusion of gravity data set. For instance, the continuous gravity data obtained from new satellite missions such as GRACE, CHAMP etc. could be handled by ANFIS to model the variations of geoid with respect to gravity variations.

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Abstract

Fuzzy set theory and later developments in uncertainty assessment have enabled us to construct more precise models valid for our requirements. In this research, Adaptive Network based Fuzzy (or in some literature Adaptive Neuro-Fuzzy) Inference System (ANFIS) has been successfully used to approximate the geoid heights both in Baden-Württemberg (Germany) and Izmir (Turkey) regions and the established fuzzy approximation models have been tested on the test points. The results have indicated its superiority against all other conventional methods and made it worthwhile to be considered in geodetic applications.

Keywords:

ANFIS · hybrid learning · fuzzy reasoning · input-output systems · membership function · geoid undulation

Zusammenfassung

Die Fuzzy Theorie und neue Bewertungen von Unsicherheiten erlauben die präzise Modellierung geodätischer Daten. In der vorliegenden Arbeit wurden unter Verwendung des adaptiven Network Fuzzy Inference System (ANFIS) Geoidhöhen in den Regionen Baden-Württemberg (Deutschland) und in Izmir (Türkei) mit Erfolg bestimmt und die Fuzzy Modelle mit vielen Testpunkten überprüft. Damit wurde gezeigt, dass das hier verwendete Verfahren bessere Ergebnisse als die herkömmlichen Verfahren liefert und für die Lösung weiterer geodätischer Probleme in Betracht gezogen werden sollte.