A new solution method to process generalized nonlinear data in building of digital scientific engineering*

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1 Introduction

How to process the spatial data is basic in the building of digital scientific projects, such as the digital earth, the digital nation, the digital city, the digital mine and so on. The data processing is the key to use the data to visualize the object. The spatial data result from many sources and have multi-dimensions, multi-types, many time states and different accuracies. The corresponding parametric estimating model to process these data is the more complex nonlinear function with respect to the random parameters and the non-random parameters. So this belongs to the generalized nonlinear data processing [1], which can be solved with the method of generalized nonlinear dynamic least squares [2]. The generalized nonlinear least squares problem is more complex than the common nonlinear least squares problem and it is more difficult to solve the former [2, 3]. Then a new solution method is put forward in the paper to separate the generalized problem into the common nonlinear least squares problem of single variable which is very easy to be solved. It mainly depends on the initial value of parameters to calculate the common nonlinear problem. Therefore the initial values closer to the true values are necessary to decrease the iterative calculating number and make the fast convergence [4, 5]. So the optimal initial values closer to the true values can firstly be obtained with the linear approaching method of nonlinear fitting model. Then the estimation values of parameters can be solved with the nonlinear least squares method. The method put forward in the paper can simplify the original high-dimensional function. In the meantime the second derivative cannot be calculated in the method. So the method can simplify the calculating difficulty and reduces the iterative number and word load. So it opens up a new way to process the generalized nonlinear data.

Suppose there is a nonlinear function

$$\mathbf{Y} = f(\mathbf{X}, \mathbf{B}) \tag{1}$$

where $\mathbf{Y} = (y_1,\ y_2, \cdots,\ y_m)^{\mathrm{T}}$ are the different kinds of spatial data with errors; $\mathbf{B} = (b_1,\ b_2, \cdots,\ b_m)^{\mathrm{T}}$ are the random parameters in nonlinear function model; and $\mathbf{X} = (x_1,\ x_2, \cdots, x_{\mathrm{n}})^{\mathrm{T}}$ are the unknown non-random parameters. The parameter $\mathbf{X}$ should appropriately be found considering the errors of $\mathbf{B}$ and $\mathbf{Y}$. So the function $\mathbf{Y} = f(\mathbf{X}, \mathbf{B})$ can fit these data to make the square sum of the fitting error be minimum. Then the problem of generalized nonlinear dynamic least squares is

$$\min F(\mathbf{X}, \mathbf{B}) = \left[\sum_{i=1}^{m} w_i (f_i(\mathbf{X}, \mathbf{B}) - y_i)^2 + v_i (b_i - \overline{b}_i)^2\right] = \sum_{i=1}^{m} \left[w_i r_i^2(\mathbf{X}, \mathbf{B}) + v_i e_i^2(\mathbf{b})\right] = \mathbf{r}(\mathbf{X}, \mathbf{B})^T\ \mathbf{W}\mathbf{r}(\mathbf{X}, \mathbf{B}) + \mathbf{e}(\mathbf{b})^T\ \mathbf{V}\mathbf{e}(\mathbf{b}) \tag{2}$$

where $\mathbf{r}(\mathbf{X}, \mathbf{B}) = (r_1(\mathbf{X}, \mathbf{B}), r_2(\mathbf{X}, \mathbf{B}), \cdots, r_m(\mathbf{X}, \mathbf{B}))^T = (f_1(\mathbf{X}, \mathbf{B}) - y_1, f_2(\mathbf{X}, \mathbf{B}) - y_2, \cdots, f_m(\mathbf{X}, \mathbf{B}) - y_m)^T$, and $\mathbf{e}(\mathbf{b}) = (e_1(\mathbf{b}), e_2(\mathbf{b}), \cdots, e_m(\mathbf{b}))^T = (b_1 - \overline{b}_1, b_2 - \overline{b}_2, \cdots, b_m - \overline{b}_m)^T$ in which $w_i \geqslant 0$ and $v_i \geqslant 0$ are the power coefficients, $\mathbf{W} = \mathrm{diag}(w_i)$, $\mathbf{V} = \mathrm{diag}(v_i)$ and $\overline{b}_i$ is the most probable value of b_i. While integrally processing the observing data with different types and accuracies, it is important to precisely determine the power coefficients of observing data in order to obtain the high accurate results. The power ratio of all data should be given, or the variances of all data should be obtained with the posterior method [6], to solve the problem.

From reference [2], he dimension of the generalized nonlinear least squares problem is m more than that of the common nonlinear least squares problem. So it is difficult to solve the former. The problem can be separated and solved which can simplify the high-power function and reduce the unknown parametric dimensions to be half.

2 Solving method

The generalized nonlinear least squares problem can be solved with the separated method. Therefore the function $F(\mathbf{X}, \mathbf{B})$ can be changed not to depend on the variable $\mathbf{B}$ with the one order necessary condition of optimal solution. Therefore the problem can be changed into a common nonlinear least squares problem only relative to the variable $\mathbf{X}$. In practice, the problem (2) can be translated into two sub-problems to be solved.

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2.1 Solution of sub-problem 1

Suppose the initial value $\overline{\mathbf{B}}$ of $\mathbf{B}$. Problem (1) is rewritten as

$$\mathbf{y}' = f(\mathbf{X}, \overline{\mathbf{B}}) \tag{3}$$

which is called sub-problem 1. At the fixed $\mathbf{B}$, parameter $\mathbf{X}$ can be solved to be satisfactory for min F $(\mathbf{X}, \overline{\mathbf{B}})$, that is

$$J_1 = \sum_{i=1}^{m} w_i(y_i' - f_i(\mathbf{X}, \overline{\mathbf{B}}))^2 = \min \tag{4}$$

The initial value $\mathbf{X}^{(0)}$ of $\mathbf{X}$ should be pre-given to solve the nonlinear least squares problem and $\mathbf{X}^{(0)}$ should be closer to the true value to reduce the iterative number and make the fast convergence. If the initial value is not good, the calculation can fail or the result is error. The selection of initial value is key to whether the iterative calculating procedure is convergent or divergent. So the initial value of $\mathbf{X}$ can be given with the linear approaching method of nonlinear fitting model [7, 8].

2.1.1 *Initial value $X^{(0)}$ of X with linear approaching method of nonlinear fitting model*

Nonlinear function (3) can be transformed into the generalized linear model [9] as

$$h(\mathbf{y}') = C_1(x_1)g_1(\mathbf{B}) + \cdots + C_n(x_n)g_n(\mathbf{B}) \tag{5}$$

where $h(\mathbf{y}')$ is the function with respect to $\mathbf{y}'$; $C_j(x_j)$ is the function with respect to the j-th parameter x_j; and $g_j(\mathbf{B})$ is the function with respect to $\mathbf{b}$. Let $\eta = h(\mathbf{y}')$, $\alpha_j = C_j(x_j)$ and $\varepsilon_j = g_j(\mathbf{B})$. Then the standard linear format is

$$\eta = \alpha_1\varepsilon_1 + \alpha_2\varepsilon_2 + \cdots + \alpha_n\varepsilon_n = \sum_{j=1}^{n} \alpha_j\varepsilon_j = S(\varepsilon, \mathbf{A}) \tag{6}$$

According to equation (4), $\hat{\mathbf{A}}$ can be obtained to solve

$$J_2 = \sum_{j=1}^{m} [\eta_j - S(\varepsilon_j, \mathbf{A})]^2 = \min \tag{7}$$

For equation (7) is nonlinear, the following linear equation group can be solved to get $\hat{\mathbf{A}} = (\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_n)$,

$$\frac{\partial j_2}{\partial x_1} = 0,\ \frac{\partial j_2}{\partial x_2} = 0, \cdots,\ \frac{\partial j_2}{\partial x_n} = 0$$

Then $x_j^{(0)} = C^{-1}(\hat{\alpha}_i)(j = 1, 2, \cdots, \mathrm{n})$ can be calculated through the inverse transformation. The parameter value $x_j^{(0)}$ is again the initial value, and then the parameter $\mathbf{X}$ should be again calculated from equation (3) with the nonlinear least squares method.

2.1.2 *Parameter $X^{(1)}$ solved from equation (3) with Gauss-Newton method[10]*

Suppose the model is $\mathbf{y}' = f(\mathbf{X}, \overline{\mathbf{B}})$, where there are n unknown parameters and $\mathbf{X}^{(1)} = (x_1^{(1)}, x_2^{(1)}, \cdots, x_n^{(1)})$. There are m samples. The initial value is $\mathbf{X}^{(0)}$ and the difference between the initial value and the true value is Δ. So

$$\begin{aligned} x_1^{(1)} &= x_1^{(0)} + \Delta_1^{(1)} \\ x_2^{(1)} &= x_2^{(0)} + \Delta_2^{(1)} \\ &\cdots \\ x_n^{(1)} &= x_n^{(0)} + \Delta_n^{(1)} \end{aligned} \tag{8}$$

which can be expressed in matrix as

$$\mathbf{X}^{(1)} = \mathbf{X}^{(0)} + \Delta^{(1)}$$

If $\Delta^{(1)}$ is determined, $\mathbf{X}^{(1)}$ is also known. In order to get $\Delta^{(1)}$, model $\mathbf{y} = f(\mathbf{X}, \overline{\mathbf{B}})$ should be expanded into Taylor series at $\mathbf{X}^{(0)}$ neglecting the second and more order terms of Δ, that is

$$f(\mathbf{X}^{(1)}, \overline{\mathbf{B}}) \approx f(\mathbf{X}^{(0)}, \overline{\mathbf{B}}) + \frac{\partial f(\mathbf{X}^{(0)}, \overline{\mathbf{B}})}{\partial x_1} \Delta_1^{(1)} + \cdots + \frac{\partial f(\mathbf{X}^{(0)}, \overline{\mathbf{B}})}{\partial x_n} \Delta_n^{(1)} \tag{9}$$

$\Delta^{(1)}$ can be solved with the common linear least squares method [11]. The residual function is

$$\begin{aligned} E &= \sum_{i=1}^{m} [w_i(y_i' - f_i(\mathbf{X}^{(1)}, \mathbf{B}))]^2 \\ &\approx \sum_{i=1}^{m} w_i \left[y_i' - \left(f(\mathbf{X}^{(0)}, \mathbf{B}) + \frac{\partial f(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_1} \Delta_1^{(1)} + \cdots + \frac{\partial f(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_n} \Delta_n^{(1)} \right) \right]^2 \end{aligned} \tag{10}$$

Then

$$\begin{aligned} \frac{\partial E}{\partial \Delta_j} = 2 \Bigg[& \Delta_1 \sum_{i=1}^{m} w_i \left(\frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_1} \cdot \frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_j} \right) + \cdots \\ & + \Delta_n \sum_{i=1}^{m} w_i \left(\frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_n} \cdot \frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_j} \right) \\ & - \sum_{i=1}^{m} w_i \left(\frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_j} (y_i' - f_i(\mathbf{X}^{(0)}, \mathbf{B})) \right) \Bigg] = 0 \end{aligned} \tag{11}$$

Let $d_{lj} = \sum_{i=1}^{m} w_i \left[\frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_1} \cdot \frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_j} \right]$,

and

$$d_{ly'} = \sum_{i=1}^{m} w_i \left[\frac{\partial f_i(\mathbf{X}^{(0)}, \mathbf{B})}{\partial x_l} (y_i' - f_i(\mathbf{X}^{(0)}, \mathbf{B})) \right]$$

in which $l = 1, 2, \cdots, \mathrm{n}$. Then we can get the linear equation group

$$\begin{aligned} & d_{11}\Delta_1^{(1)} + d_{12}\Delta_2^{(1)} + \cdots + d_{1n}\Delta_n^{(1)} = d_{1y'} \\ & \cdots \\ & d_{n1}\Delta_1^{(1)} + d_{n2}\Delta_2^{(1)} + \cdots + d_{nn}\Delta_n^{(1)} = d_{ny'} \end{aligned} \tag{12}$$

The equation group is solved to get $\Delta^{(1)} = (\Delta_1^{(1)}, \Delta_2^{(1)}, \cdots, \Delta_n^{(1)})$. Then $\Delta^{(1)}$ is substituted for equation

(8) and $\mathbf{X}^{(1)}$ can be solved. $\mathbf{X}^{(1)}$ is much more closer to the true value than $\mathbf{X}^{(0)}$.

2.1.3 Final estimation of X

Repeat the above procedures and calculate $\Delta^{(2)}, \mathbf{X}^{(2)}, \cdots, \Delta^{(K)}$ and $\mathbf{X}^{(K)}$ until $\|\Delta^{(K)}\|$ is less than the permitted error. Now $\mathbf{X}^{(K)}$ is the final estimation value of $\mathbf{X}$.

2.2 Calculation of min $f(\mathbf{X}^{(K)},\mathbf{B})$ at the solution $\mathbf{X}^{(K)}$

Now sub-problem 2 is

$$\mathbf{y}'' = f(\mathbf{X}^{(K)}, \mathbf{B}) \tag{13}$$

$\mathbf{B} = (b_1, b_2, \cdots, b_m)$ can be solved to compute

$$\min F(\mathbf{X}^{(K)}, \mathbf{B}) = \sum_{i=1}^{m} \left[w_i(\mathbf{y} - f(\mathbf{X}^{(K)}, \mathbf{B}))^2 + v_i(b_i - \overline{b}_i)^2 \right] \tag{14}$$

at the fixed $\mathbf{X}^{(K)}$. The initial value $\mathbf{B}^{(0)}$ of $\mathbf{B}$ is firstly estimated with the linear approaching method of nonlinear fitting model. Equation (13) can properly be translated into the generalized linear format as

$$\mathrm{h}(\mathbf{y}'') = \mathrm{C}'_1(b_1)g'_1(\mathbf{X}^{(K)}) + C'_2(b_2)g'_2(\mathbf{X}^{(K)}) + \cdots + C'_m(b_m)g'_m(\mathbf{X}^{(K)}) \tag{15}$$

where $h(\mathbf{y}'')$ is the function with respect to $\mathbf{y}''$; $C'_i(b_i)$ is the function with respect to the i-th parameter b_i; and $g'_i(\mathbf{X}^{(K)})$ is the function of $\mathbf{X}$. Let $\eta' = h(\mathbf{y}'')$, $\alpha'_i = C'_i(b_i)$, $\xi' = g'_i(\mathbf{X}^{(K)})$ and $\eta' = \sum_{i=1}^{m} \alpha'_i \xi'_i = S'(\xi', \mathbf{A}')$. From equation (14), $\mathbf{A}'$ can be solved to make

$$J_3 = \sum_{i=1}^{m} \left[w_i(\eta'_i - S'(\xi'_i, \mathbf{A}'))^2 + v_i(b_i - \overline{b}_i)^2 \right] = \min \tag{16}$$

Based on the condition of extreme value, we can get

$$\frac{\partial J_3}{\partial b_1} = 0, \ \frac{\partial J_3}{\partial b_2} = 0, \cdots, \ \frac{\partial J_3}{\partial b_m} = 0$$

After $\mathbf{A}'$ is obtained, $\mathbf{B}^{(0)} = (b_1^{(0)}, b_2^{(0)}, \cdots, b_m^{(0)})$ is also gotten through the inverse transformation.

$\mathbf{B}^{(0)}$ being the initial value, equation (14) can again be solved with the Gauss-Newton method to compute the parameter $\mathbf{B}$. Then

$$(d_{11} + v_1)\Delta_{b_1}^{(1)} + \cdots (d_{1m} + v_m)\Delta_{b_m}^{(1)} = d_{ly''}$$

$$\cdots$$

$$(d_{m1} + v_1)\Delta_{b_1}^{(1)} + \cdots (d_{mm} + v_m)\Delta_{b_m}^{(1)} = d_{my''}$$

Solve the above the linear equation group to get $\Delta_b^{(1)}$ from which $\mathbf{B}^{(1)} = (b_1^{(1)}, b_2^{(1)}, \cdots, b_m^{(1)})$ is computed. $\mathbf{B}^{(1)}$ is more closer to the true value than $\mathbf{B}^{(0)}$.

Repeat the above procedures to get $\Delta_b^{(2)}, \mathbf{B}^{(2)}, \cdots, \Delta_b^{(K)}, \mathbf{B}^{(K)}$ until $\|\Delta_b^{(K)}\|$ is less than the permitted error. Now $\mathbf{B}^{(K)}$ is the final result.

The solution method put forward in the paper can reduce the dimension of the nonlinear problem and search iteratively the initial value with the optimal method. So the method can reduce the iterative number and the computing load, and make the fast convergence. In the meantime, it is easy to compute the generalized nonlinear least squares problem with the method. The method can extend the application of the generalized nonlinear least squares method in data processing. The method is suitable to solve the high-dimensional problem and has the bright application prospects.

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Abstract

Data, including the spatial data and the non-spatial data, are the basement of all digital scientific engineering projects, such as the digital earth, the digital nation, the digital city, the digital mine and so on. The spatial data has the characteristics of many sources, multi-dimensions, multi-types, many time states and different accuracies. The spatial data processing firstly must be made before using these data. The parametric estimating model to process the data is commonly the more complex nonlinear model including the random parameters and the non-random parameters together. This belongs to the generalized nonlinear data processing, which should be solved by the generalized nonlinear dynamic least squares theory. The generalized nonlinear least squares problem is more complex than the common nonlinear least squares problem and it is more difficult to solve the former. Then a new solution method is put forward in the paper to separate the generalized problem into the common nonlinear least squares problem of single variable which is very easy to be solved. It mainly depends on the initial value of parameters to calculate the common nonlinear problem. Therefore the initial values closer to the true values are necessary to decrease the iterative calculating number and make the fast convergence. So the optimal initial values closer to the true values can firstly be obtained with the linear approaching method of nonlinear fitting model. Then the estimation values of parameters can be solved with the nonlinear least squares method. The method put forward in the paper can simplify the original high-dimensional function. In the meantime the second derivative cannot be calculated in the method. So the method can simplify the calculating difficulty and reduces the iterative number and word load.

Keywords: Generalized nonlinear dynamic least squares problem, nonlinear fitting, separating solution

BUCHBESPRECHUNG

Manfred Bauer

Vermessung und Ortung mit Satelliten

5., neu bearbeitete und erweiterte Auflage, GPS und andere satellitengestützte Navigationssysteme, Herbert Wichmann Verlag, Heidelberg, 2003, 392 Seiten, kartoniert.
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Um der Idee seines Buches, *die Grundprinzipien satellitengestützter Vermessung und Ortung darzustellen*, weiterhin gerecht werden zu können, gibt es neben Manfred Bauer in der neuen Auflage seines Buches mit Lambert Wanninger erstmals einen Co-Autor. Darüber hinaus sind an der aktuellen Auflage des Buches noch eine Reihe weiterer Experten dieser Thematik beteiligt.

Für die neue Auflage des Buches hat dies eine Reihe von Konsequenzen, die sich für den Leser nicht nur im veränderten Layout der Ausgabe niederschlagen. So gliedert sich das Buch nunmehr in insgesamt acht Kapitel (bisher sieben) und drei Anhänge. Darauf folgen Glossar, Abkürzungsverzeichnis, Literaturverzeichnis und Internetverweise. Abgerundet wird das Buch schließlich mit dem Sachwörterverzeichnis, welches in der letzten Ausgabe noch unter dem Namen Stichwortverzeichnis zu finden war. Das Inhaltsverzeichnis der neuen Auflage ist übersichtlicher geworden, da die Überschriften nur noch bis zur dritten Überschriftenebene dargestellt werden.

In Kapitel 1 *Einführung* wird – wie gewohnt – allgemein in die Thematik GPS eingeführt. Darauf folgt die Erläuterung und Darstellung der *Theoretische Grundlagen* in Kapitel 2. Hier findet sich als neues Unterkapitel 2.4 *Überführung von ellipsoidischen Höhen in Gebrauchshöhen* eine ausführliche Abhandlung zum Thema GPS und Höhen, was in dieser Form in den bisherigen Ausgaben noch fehlte. Kapitel 3 *GPS* wurde gegenüber der 4. Auflage dahingehend verändert, dass das Unterkapitel 10 *Relative GPS-Positionierung* gründlich von Herrn Wanninger aufgearbeitet wurde und die Unterkapitel 13 und 14 aus dem Kapitel 3 herausfallen und als neues Kapitel 8 *Ortung und Vermessung mit Satelliten in der Praxis* eingeführt werden. Das System Glonass wird im gleichnamigen Kapitel 4 dargestellt. Neu sind hier die Inhalte in den Unterkapiteln 4.7 und 4.8, die teilweise erweitert wurden und in denen sich vor allem das ehemalige Kapitel 5 *GNSS – Das Globale Navigationssatellitensystem der Zukunft?* wiederfindet, welches folglich nicht mehr als eigenes Kapitel existiert. Weitere satellitengestützte Ortungssysteme werden nun in Kapitel 5 behandelt. Kapitel 6 beschäftigt sich ausschließlich mit dem System *INMARSAT.* Die Unterscheidung in die Unterkapitel 6.2 *INMARSAT*-Dienste und 6.3 *INMARSAT und Satellitennavigation* ist eher unglücklich, da sich die beiden Unterkapitel inhaltlich sehr ähnlich sind. Mit Kapitel 7 *Europäische Aktivitäten zur Entwicklung eines zivil kontrollierten GNSS* trägt der Autor den aktuellen Entwicklungen der Europäischen Gemeinschaft auf dem Sektor Satellitennavigation, nämlich der Entwicklung eines eigenen, von den USA unabhängigen, Satellitennavigationssystems (GALILEO) Rechnung. Etwas beiläufige

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