Check on Antenna Phase Center Three-dimensional Bias of GPS Receiver

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On the basis of the relative positioning method, using the precise leveling, a new method was put forward to examine the antenna phase center three-dimensional bias of GPS receiver with the geometric relation of the antenna phase centers and the mini baseline in the paper. Practical cases indicate that the method is very reliable and precise, and is fit for the examination of phase center bias in the field.

1 Introduction

With the character of fast work, highly precise, flexible controlling and work in whole day, GPS is widely applied in surveying and mapping field [LIU, et al, 1996; ZHOU, et al, 1999; LI, 1996]. But the precision of elevation is two or three times inferior to that of horizontal position in GPS surveying. The key reason is that the tropospheric correction is not enough accurate. But the antenna phase center bias is another important factor also to impact on GPS precise, especially mixing the different forms' antenna to work together, which would produce the bias of relative elevation between stations up to centimeter order, even decimeter order. On the other hand, the scale error is about 0.0153 10^{-6} using the same type antennas in GPS surveying. The effect of antenna phase center bias on horizontal position is up to several millimeters. So it cannot be neglected in precise positioning surveying, for example, deformation monitoring for dam and high building, displacement and deformation monitoring in mineral area, surface sedimentation monitoring in city, diastrophism monitoring and so on.

Antenna phase center is the electric center of microwave antenna, whose design is consistent to the geometrical center of GPS antenna in theory. But there exists bias between antenna phase center and geometrical center because of produce level of antenna, the incident direction of GPS signal and so on, and the bias is called as antenna phase center bias. There are three methods commonly used to check the antenna phase center bias of GPS receiver, namely as revolving antenna method, relative positioning method and exchanging antenna method [CAI, 2000; CHEN, et al, 2000]. Revolving antenna method needs the microwave antenna instrument and microwave black-box to measure the phase center inside. Therefore the method has the shortage of complex, expensive equipments, expensive examination, and more time, and the method is not fit for examination of bias in the field. Relative positioning method can be made outside anytime, but it can only measure two-dimensional bias of antenna phase center, not the bias in elevation. Exchanging antenna method can be used to measure the vertical bias of antenna phase center, but not the horizontal bias. In the meantime, it is not suit for the GPS receiver that its mainframe is attached with its antenna. Hence, we can examine the antenna phase center three-dimensional bias of GPS receiver that its mainframe is apart with its antenna with combined method of exchanging antenna and relative positioning [CAI, 2000], but it must be made step by step, spend much time and has different precision.

On the base of relative positioning method, a method that can be used to check the phase center three-dimensional bias is put forward according to precision leveling and the geometrical relationship of antenna phase center and geometry center in this paper. The method can be used to measure the horizontal and height bias of antenna phase center in the meantime, and is fit for all kinds of GPS receiver in surveying in the field.

2 Theory and method

In an area of wide view and no interference of strong Hertzian wave and reflected environment we can choose a mini baseline of 1meter to 10 meters in length to check on antenna phase center bias in the field. Setting up GPS receivers on each end-point of the mini baseline and keeping the antenna level rigidly, we can choose a corresponding time of PDOP≤5 to observe the baseline. The phase center commonly is the average phase center, while the instantaneous phase center changes with the time. So much more time is needed to observe the baseline in order to get more steady average phase center, about 1.5 hours.

Showed in Fig.1, *a* and *b* are antennas GPS receivers, whose corresponding geometrical centers are O_a and O_b respectively. Centering by O_a, the direction of meridian standing for *x* axis, and *z* axis pointing to zenith *y* axis and *x*, *z* axis are formed a left-hand system which is called as the station-centered right-angle coordination system O_axyz. Otherwise, centering by O_b and *x' y'* and *z'* parallel to *x*, *y* and *z* respectively we can found a local coordination system $O_bx'y'z'$. Then (x_a, y_a, z_a) and (x_b, y_b, z_b) are the phase center biases of antenna *a* and *b* respectively when the antennas' north marks point to direction of *x* axis.

Supposing O_a's geodetic coordination is (L_a, B_a, H_a) in

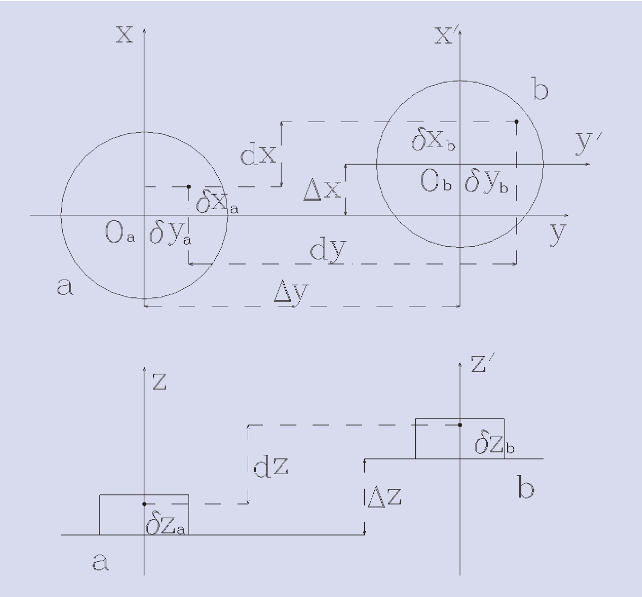

Fig. 1: Sketch map of test on antenna phase center three-dimensional Bias of GPS Receiver

ellipse of WGS-84 and its WGS-84 right-angle coordination is (X_a, Y_a, Z_a), then the coordination (X, Y, Z) in station-centered coordination system $O_a xyz$ is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin B_a \cos L_a & -\sin B_a \sin L_a & \cos B_a \\ -\sin L_a & \cos L_a & 0 \\ \cos B_a \cos L_a & \cos B_a \sin L_a & \sin B_a \end{bmatrix} \begin{bmatrix} X - X_a \\ Y - Y_a \\ Z - Z_a \end{bmatrix} \quad (1)$$

in the right-angle coordination system of WGS-84.

Assuming the mini baseline vector in WGS-84 is (ΔX, ΔY, ΔZ), and the corresponding baseline in station-centered right-angle coordination system is (dx, dy, dz), from formula (1) we can get

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} -\sin B_a \cos L_a & -\sin B_a \sin L_a & \cos B_a \\ -\sin L_a & \cos L_a & 0 \\ \cos B_a \cos L_a & \cos B_a \sin L_a & \sin B_a \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \quad (2)$$

2.1 Check of horizontal bias of antenna phase center

Because there are six unknown parameters including horizontal part of a baseline and two horizontal biases of antennas phase centers in a baseline, six observations is needed at least. Fixing antenna a to make the north-mark point to the direction of x axis, revolving antenna b by a settled θ in clockwise and $n = \frac{360°}{\theta}$ must be an integer, seeing in Fig. 1, the horizontal part of phase center of antenna b after the ith rotation is

$$\begin{bmatrix} \delta x_{b_i} \\ \delta x_{b_i} \end{bmatrix} = \begin{bmatrix} \cos i\theta \delta x_b - \sin i\theta \delta y_b \\ \sin i\theta \delta x_b + \cos i\theta \delta y_b \end{bmatrix} \quad (i = 0, 1, 2, \ldots, n-1) \quad (3)$$

Then the ith baseline vector in WGS-84 can be transformed to the horizontal baseline part in station-centered horizon coordination system by formula (2). Therefore, the relationship among the horizontal part, the baseline in the sight of geometrical center and two antennas phase centers is

$$\begin{bmatrix} dx_i \\ dy_i \end{bmatrix} = \begin{bmatrix} \Delta x - \delta x_a + \cos i\theta \delta x_b - \sin i\theta \delta y_b \\ \Delta y - \delta y_a + \sin i\theta \delta x_b + \cos i\theta \delta y_b \end{bmatrix} \quad (i = 0, 1, 2, \ldots, n-1) \quad (4)$$

Fixing antenna b to make its north-mark point to the direction of x axis and revolving antenna a by a settled θ in clockwise, then we can get the horizontal part of antenna phase center after the jth rotation

$$\begin{bmatrix} \delta x_{a_i} \\ \delta y_{a_i} \end{bmatrix} = \begin{bmatrix} \cos j\theta \delta x_b - \sin j\theta \delta y_b \\ \sin j\theta \delta x_b + \cos j\theta \delta y_b \end{bmatrix} \quad (j = 1, 2, \ldots, n-1) \quad (5)$$

Then the (n+j–1)th baseline vector in WGS-84 can be transformed to the horizontal baseline part in station-centered horizon coordination system by formula (2). Therefore, the relationship among the horizontal part, the baseline in the sight of geometrical center and two antennas phase centers is

$$\begin{bmatrix} dx_{j+n-1} \\ dy_{j+n-1} \end{bmatrix} = \begin{bmatrix} \Delta x + \delta x_b - \cos j\theta \delta x_b + \sin j\theta \delta y_a \\ \Delta y + \delta y_b - \sin i\theta \delta x_a - \cos j\theta \delta y_a \end{bmatrix} \quad (j = 1, 2, \ldots, n-1) \quad (6)$$

Hence, there are 2n–1 measurements and 4n–2 equations. So the error equation represents in matrix as

$$\boldsymbol{V} = \boldsymbol{B} \quad - \boldsymbol{l} \quad (7)$$

where, $= [dx_a \ dy_a \ dx_b \ dy_b \ \Delta x \ \Delta y]^T$ is the vector of unknown parameters; $\boldsymbol{l} = [\ x_1 \ \ y_1 \ \ x_2 \ \ y_2 \ \ldots \ dx_{2n-1} \ dy_{2n-1}]^T$ is the vector of observations; $\boldsymbol{V} = [v_{x_1} \ v_{y_1} \ v_{x_2} \ v_{y_2} \ \ldots \ v_{x_{2n-1}} \ v_{y_{2n-1}}]^T$ is the vector of observation correction; $\boldsymbol{B} = [\boldsymbol{b}_1 \ \boldsymbol{b}_2 \ \ldots \ \boldsymbol{b}_{4n-2}]^T$ is the designed matrix, in which $\boldsymbol{b}_p$ (p = 1, 2, …, 4n–2) is a column vector and

$$\boldsymbol{b}_{p=} \begin{cases} \begin{bmatrix} -1 & 0 & \cos i\theta & -\sin i\theta & 1 & 0 \end{bmatrix}^T, & \bullet\ p = 2i+1, i = 0, 1, \ldots, n-1 \\ \begin{bmatrix} 0 & -1 & \sin i\theta & \cos i\theta & 0 & 1 \end{bmatrix}^T, & p = 2i, i = 0, 1, \ldots, n-1 \\ \begin{bmatrix} -\cos j\theta & \sin j\theta & 1 & 0 & 1 & 0 \end{bmatrix}^T, & p = 2n+2j-1, j = 1, 2, \ldots, n-1 \\ \begin{bmatrix} -\sin j\theta & -\cos j\theta & 0 & 1 & 0 & 1 \end{bmatrix}^T, & p = 2n+2j, j = 1, 2, \ldots, n-1 \end{cases} \quad (8)$$

According to the least squares theory, solving equation (7), we can get

$$= (\boldsymbol{B}^T \boldsymbol{PB})^{-1} \boldsymbol{B}^T \boldsymbol{Pl} \quad (7)$$

where, $\boldsymbol{P}$ is the weight matrix of observations,

$$\boldsymbol{P} = \begin{bmatrix} P_1 & & & \\ & P_2 & & \\ & & 0 & \\ & & & P_{4n-2} \end{bmatrix}, \text{ in which}$$

$$P_k = \begin{cases} 0, & \text{while } t < 0.5 \\ \frac{t}{1.5}, & \text{the other} \end{cases}$$, t is the time interval of observation in unit of hour.

So we can get the unit weight mean error

$$m_0 = \pm \sqrt{\frac{\boldsymbol{V}^T \boldsymbol{PV}}{4n-8}} \quad (10)$$

and the coefficient matrix of unknown parameter is

$$\boldsymbol{Q} \quad = (\boldsymbol{B}^T \boldsymbol{PB})^{-1} \quad (11)$$

While examining the horizontal bias of GPS receiver antenna phase center in the field, there are two cases commonly used, which are $\theta = 90°$ and $\theta = 45°$. They have the merit of grasping and manipulating easily and enough redundant measurements in practice.

2.1.1 The case of $\theta = 90°$

We can rotate antenna a in the clockwise by the angle of 90°, 180° and 270° from a_1 (x_a, y_a) to a_2 (– y_a, x_a), a_3 (– x_a, – y_a) and a_4 (y_a, – x_a) respectively in the field. In the same way, we can rotate antenna b in the clockwise by the angle of 90°, 180° and 270° from b_1 (x_b, y_b) to b_2 (– y_b, x_b), b_3 (– x_b, – y_b) and b_4 (y_b, – x_b) respectively. Therefore there are seven observing stages and fourteen equations to solve the unknown parameters.

2.1.2 the case of $\theta = 45°$

At this case, the revolving angle of antenna is 45° every time. Then there are fifteen observing stages and thirty equations. It is difficult to be solved and need twice more time, but its precision and reliability all are better than the case of $\theta = 90°$.

2.2 Examining of vertical bias of antenna phase center

According to the geometric relationship of two antennas showed in Fig. 1, in theory we can get

$$z_b - z_a = dz - ¢z \qquad (12)$$

When examining the horizontal part of antenna phase center bias, there are $2n$–1 baseline part ¢ z in the coordination system of WGS-84 in total which can be transformed to the vertical part dz of station-centered horizon coordination system by formula (2). In formula (12), if knowing ¢ z precisely, we can calculate the difference of vertical part between two antenna phase center biases. Thus, at the same time of checking horizontal part of phase center bias, we should measure the difference ¢ z in elevation between two antennas path-restrained boards with precise leveling, and perform in one or two grade leveling with Zeiss Ni004, Koni 007 or Wild N_3 equipment to get $2n$–1 measurement. So the ith result is

$$¢ z_i = dz_i - ¢ z_i \qquad (13)$$

Taking its average value we get the last result

$$\overline{\Delta\delta z} = \frac{1}{2n-1}\sum_{i=1}^{2n-1} \Delta\delta z_i = \frac{1}{2n-1}\sum_{i=1}^{2n-1}(dz_i - \Delta z_i) \qquad (14)$$

Then the checking mean error of difference in vertical part of two antenna phase center biases is

$$m_{\Delta\delta z} = \pm\sqrt{\frac{[vv]}{2n-2}} \qquad (15)$$

where, v_i = ¢ z_i – $\overline{¢\ z}$.

3 Examples

We test on three groups of antenna phase center biases with two receivers of Zeiss RD24, two receivers of RS12 and two receivers of RD24 and RS12, respectively. The selected mini baseline is about five meters, whose direction is near to east-west. Examining the horizontal heft of phase center bias, we perform with revolving angular θ = 90°, and the observing interval of 1.5 hours every time. Performing precise leveling with Koni 007 level and measuring the instrument height five times pre and post measurement respectively, we can get their difference in each other less then or equal to 1 millimeter and take their average value as the last result. The three group's results are showed in table 1 and Fig. 2.

Table 1: The three-dimensional examining results of GPS receiver phase center bias (unit in millimeter)

x_a	y_a	x_b	y_b	$\overline{¢\ z}$	memo
1.32° ± 0.27	1.60° ± 0.27	1.25° ± 0.27	1.54 °± 0.27	2.26° ± 0.87	Two Zeiss RD24
–2.42° ± 0.31	–1.54° ± 0.31	–2.29° ± 0.31	–1.40 °± 0.31	2.83° ± 0.94	Two Zeiss RS12
1.40° ± 0.49	1.92° ± 0.49	–2.08° ± 0.49	–1.37° ± 0.49	4.36° ± 1.13	a is Zeiss RD24 and, b is Zeiss RS12

From table 1, we can know that the horizontal bias of phase center of GPS receiver which is of the same type and produced in same time is close to homology, and the difference in vertical part is sub-millimeters. Those of the different type receiver antenna have much difference, especially, when its azimuth difference of phase center bias is larger, and the difference of its vertical heft will be up to millimeters. Therefore, the effect of antenna phase center bias can be neglected in general engineering surveying. But, we must consider the effect of antenna phase center bias in precise engineering surveying and high precise positioning surveying. What is more, the effect of antenna phase center bias should be considered to improve the precision in GPS leveling.

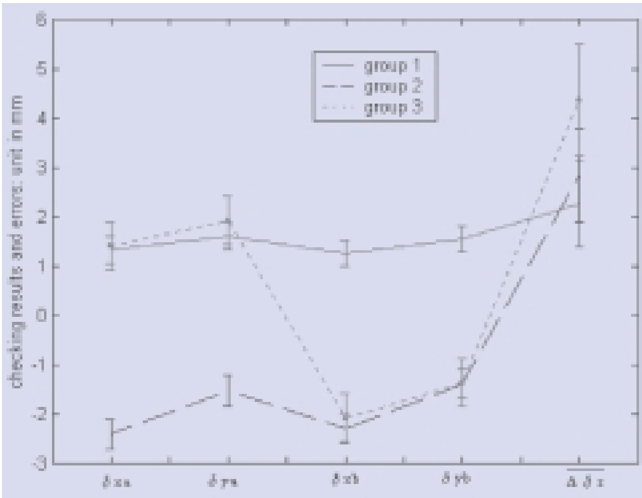

Fig. 2: Checking results and errors

4 Conclusion

On the basis of the relative positioning method, a new method is put forward to examine the antenna phase center three-dimensional bias of GPS receiver with the geometric relationship of the antenna phase centers in the paper. Practical cases indicate that the method is very reliable and precise, and is fit for the examination of phase center bias in the field. Meanwhile, the examples show that phase center bias of the same type GPS receiver is close to homology, and the difference in vertical heft is up to millimeters. Those of the different type receiver antenna have much difference.

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