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Analysis of including GPS measurements into terrestrial systems

Bei der Erneuerung des geodätischen Festpunktfeldes in der Republik Kroatien werden GPS-Messungen verwendet. Zur Einbindung dieser Messungen in das vorhandene Punktfeld werden zwei Alternativen untersucht, mit und ohne Beachtung der Nachbarschaft.

1 Introduction

Geodetic – spatial system of the Republic of Croatia has been impoverished in the last seventy years with respect to its structure and program by numerous inconvenient measures. The establishment of the independent Republic of Croatia, the definition of market oriented economic system with the land becoming an economic subject have made difficulties caused by not keeping the high quality real estate register become noticeable.

Since the works on establishing trigonometric network at the territory of the Republic of Croatia have been carried out in a long period of time, under various political and economic conditions and circumstances, the co-ordinates of points have been determined in several co-ordinate systems and with different degree of accuracy. Poor physical preserved condition and unreliability of co-ordinates of geodetic control points, and insufficiently defined criteria of accuracy enforce the need for their renewal by applying modern satellite technologies in the combination with classical terrestrial measurements. In order to improve the quality of geodetic basis, and to include all measurements for the present cadastral plans into the existing trigonometric network parallelly with their continuous maintenance, these problems are to be solved within the scope of two models of including GPS measurements into the reference state co-ordinate system, i.e. a model of including without preserving the neighbourhood and with preserving the neighbourhood. These models have been applied, tested and analysed on a test network encompassing broader area of the city Samobor (in the surroundings of Zagreb).

2 Transformation of co-ordinates

The results of GPS measurements are the co-ordinates or co-ordinate differences defined in the reference system ETRS'yy (on the ellipsoid WGS'84) that are not defined in the direct relationship with the co-ordinate systems of the state survey. In order to use and maintain the existing cadastral plan, the including of GPS measurements into the state co-ordinate system is solved by means of Helmert's spatial transformation using identical points in both systems or by means of previously computed transformation parameters. Comparing the co-ordinates (valid, state and those obtained by Helmert's transformation) of identical points we obtain the assessment of accuracy and homogeneity of the existing network. The residuals for each axis can run up to 5 cm maximum in the identical points according to (Niemeier 1985, 1987). The points that have bigger residuals are excluded as inappropriate for computing transformation parameters. It is recommendable to explore the cause of bigger differences.

Before transforming co-ordinates from ETRS'yy (WGS'84) into the state co-ordinates system, one should have in mind the fact that the heights relate to one reference surface, and position co-ordinates to the other. It is therefore necessary to know the elevation difference between these reference surfaces, and it is yielded exclusively by geoid undulations. Introducing geoid undulations during the transformation of co-ordinates is a correct procedure from the scientific and professional point of view. Their influence on the determination of heights is rather remarkable, but not worth mentioning when speaking of position co-ordinate of points (y and x) (Džapo, 1998).

3 Results of including the new GPS network into the State Co-ordinate System

The theoretical knowledge about including GPS networks into the terrestrial systems has been applied and also checked on the network of the city Samobor located in the mountain and forest region. The existing geodetic network encompassing the area of 150 km² was established in the period from 1937 to 1940, renewed in 1948 and finally completed in 1959/1960. It leans on four 1. order trigonometric points and is adjusted by the method of conditional measurements. Apart from the four 1. order points, there were also 150 II., III., and IV. order trigonometric points established and determined. A great part (about 75 %) of these points is ruined today, and many of them have lost their function due to the expansion of the city. Poor preserved condition of these points initiated the renewal and condensation of the above-mentioned trigonometric network. After new stabilisation of some points in the old network has been done and taking the economic needs of the city Samobor into account, a new geodetic basis has been established. GPS measurements have been carried out using static method and Trimble receiver.

Table 1: The residuals after the transformation of co-ordinates

Point	dy	dx	dh
Δ9	+0,012	-0,011	+0,094
Δ10	+0,003	-0,011	-0,022
Δ28	-0,012	+0,004	+0,024
Δ29	-0,004	-0,049	+0,076
Δ31	+0,053	+0,019	-0,014
Δ32	+0,010	+0,009	-0,070
Δ34	+0,046	0,000	+0,070
Δ46	+0,013	-0,002	-0,022
Δ48	-0,029	+0,022	+0,006
Δ124	+0,008	+0,005	+0,084
Δ125	-0,043	-0,025	+0,095
Δ152	-0,030	+0,010	-0,008

Table 2: Differences between directly measured lengths and those obtained from co-ordinates

From – to	Δ [mm] D_{meas} - $D_{co\text{-}or.}$	Relative accuracy Δ/D_{meas}
48-1304	- 39	1:18000
48-1305	- 42	1:15000
48-1306	+38	1:11000
48-1307	+40	1:13000
48-1256	+34	1:43000
9-1084	-8	1:100000
9-1085	+13	1:39000
9-1087	-11	1:18000
9-1088	+22	1:13000
152-1259	+29	1:14000
152-1260	+34	1:17000
29-1059	-5	1:108000
29-1060	-23	1:49000
29-1093	+58	1:4000
31-1281	-39	1:23000
New points		
1237-1123	+14	1:85000
1237-1124	- 6	1:250000
1123-1126	- 5	1:150000
1087-1063	+ 6	1:84000
1087-1088	+ 9	1:42000
1131-1133	+ 8	1:74000
1082-1087	- 6	1:185000
1227-1229	- 7	1:250000
1130-1133	- 3	1:340000
1119-1120	- 5	1:52000

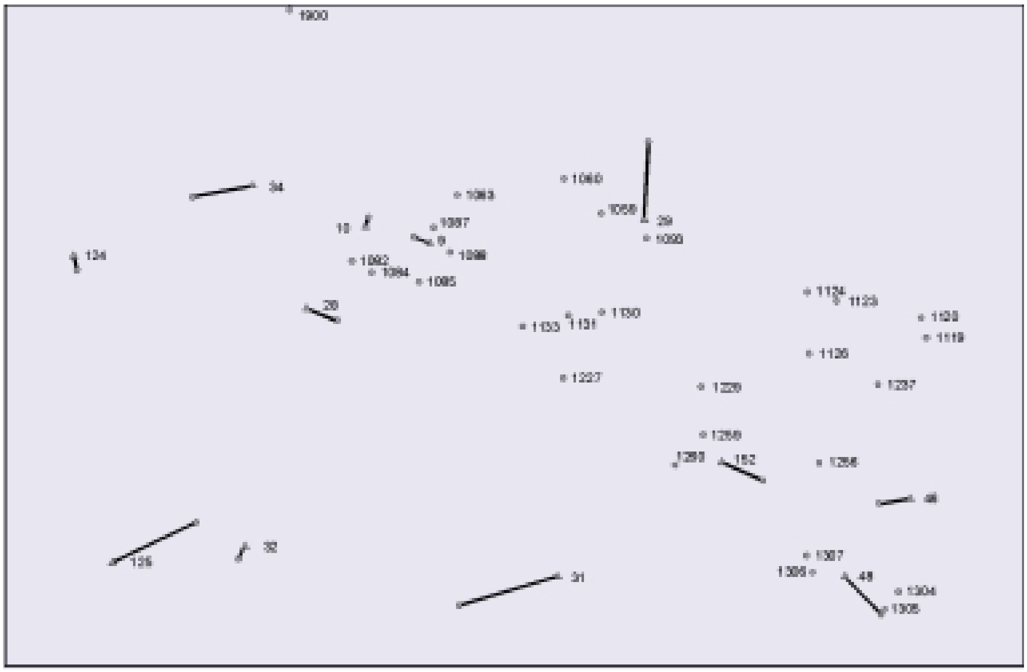
The renewed network has today 320 points with 37 among them being old trigonometric points. It has been planned to set up the filed of points for surveying the details for the purpose of producing new and maintaining the existing cadastral plans.

3.1 The results of including according to the model without preserving the neighbourhood

For the purpose of including GPS measurements into the valid State Co-ordinate System, as it had been mentioned in (Niemeier, 1987), only the part of GPS measurements without datum has been suggested to be used. The usage of GPS information part without datum and familiarity with transformation parameters is sufficient to have GPS measurements included into the reference State Co-ordinate System.

For the purpose of computing transformation network parameters, 27 trigonometric points from the first to the fourth order have been used. After carrying out the transformation procedure from ETRS’89 (WGS’84) into the Gauss-Krüger co-ordinate system, there were 12 points left that meet the determined criteria – tolerance up to 5 cm (Table 1, Fig. 1). Since the data about the geoid undulations for the territory of the city Samobor have been obtained from a small number of points and residuals are somewhat bigger by height that the accepted 5 cm, at the point in the mountain part of the network, because of objectively possible higher accuracy in determining the heights with GPS measurements. Graphic presentation of co-ordinate deviation after transformation is given on Fig. 1.

The size of the residuals depends first of all on the network quality being the consequence of applied measuring and computing procedures, and then on the order of trigonometric points.



Δ – existing trigonometric points, – GPS points (Scale: 1:60 000; Deviation scale: 1:2)

Fig. 1: Deviations on trigonometric points after the transformation (only along the axes x and y)

In order to test the applied method of including, and to evaluate the quality of the established geodetic network at the same time, the lengths among a few selected points have been measured directly with electrooptical distance meter and compared with the lengths obtained from the computed co-ordinates (y, x) of points in the State Co-ordinate System (Table 2). Directly measured lengths have been corrected for the atmospheric correction, reduced to the surface of the reference ellipsoid and deduced to the projection surface.

It is to be seen from the table 2 that the differences of directly measured lengths and those obtained from the co-ordinates are significant between the old trigonometric points and newly determined points, and the relative accuracy is small. The differences among newly determined points are small, and relative accuracy is high. Hence, such network is not homogeneous, and it has large tensions.

It is obvious that this way of including is not acceptable for the networks of lower quality, i.e. those networks having big residuals fully expressed in the relationship between the old (valid) trigonometric points and their neighbouring newly determined points. It would therefore be very reasonable to retain the old, valid co-ordinates of the existing trigonometric points, and to take their values computed by means of transformation for new points. This way of including without preserving the neighbourhood can be applied, as it has been stated in (DENKER, 1989), for geodetic networks having positional accuracy of trigonometric points from 1 to 2 cm.

3.2 The results of including according to the model with preserving the neighbourhood

Since the presented model of including the GPS network into our State Co-ordinate System did not give satisfactory results because of low positional accuracy of the trigonometric network, it was necessary to find the solution in the model implying the preserved condition of neighbourhood that should keep good relative relationships among neighbouring points.

The model of including with preserving the neighbourhood helps in reducing the residuals at datum points to zero, and the new points, as well as some old trigonometric points get corrections depending on the size of residuals and the distances from datum points. In order to realise the including with preserving the neighbourhood, as it has been stated in (NIEMEIER, 1987), one should introduce the co-ordinates of datum points as the observation having great weight and also

Table 3: Differences between directly measured lengths and those obtained from the co-ordinates according to the model with preserving the neighbourhood

From – to	Δ [mm] $D_{meas} - D_{co\text{-}or.}$	Relative accuracy Δ/D_{meas}
48-1304	-12	1:60000
48-1305	-9	1:72000
48-1306	+10	1:41000
48-1307	+12	1:46000
48-1256	+29	1:51000
9-1084	-8	1:104000
9-1085	+7	1:73000
9-1087	-2	1:103000
9-1088	+5	1:56000
152-1259	+7	1:59000
152-1260	+10	1:58000
29-1059	0	-
29-1060	-10	1:113000
29-1093	+4	1:59000
31-1281	-13	1:68000
New point, with "preserved neighbourhood"		
1237-1123	+19	1:63000
1237-1124	-1	1:148000
1123-1126	-2	1:376000
1087-1063	+10	1:50000
1087-1088	+8	1:47000
1131-1133	+10	1:59000
1082-1087	-11	1:102000
1227-1229	+6	1:294000
1130-1133	+3	1:341000
1119-1120	-1	1:260000

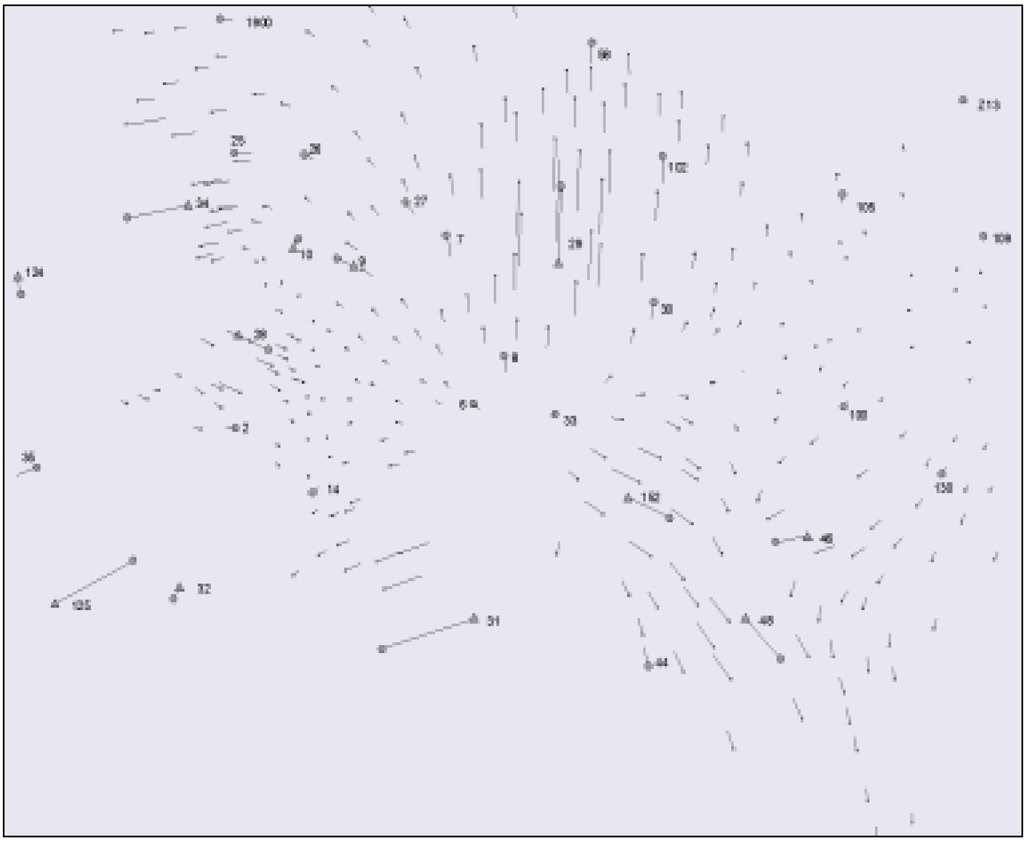


Fig. 2: Including of GPS measurements with preserved condition of the neighbourhood

introduce additional correlation between the neighbouring points.

The result achieved by using the above mentioned model of including is shown in the previous table where the differences between directly observed lengths and those computed from co-ordinates are presented (Table 3).

After the application of this including model, all datum points have remained fixed, i.e. they have retained their old co-ordinates, and new points, as well as some old trigonometers, have got corrections on the basis of linear correlation function and correlation distance of 3 km (BÄHR/RICHTER, 1975) which is to be seen on Fig. 2.

The presented model of including accompanied by preserving the neighbourhood is realised completely by means of a compute program "PANDA" (GeoTec, 1996).

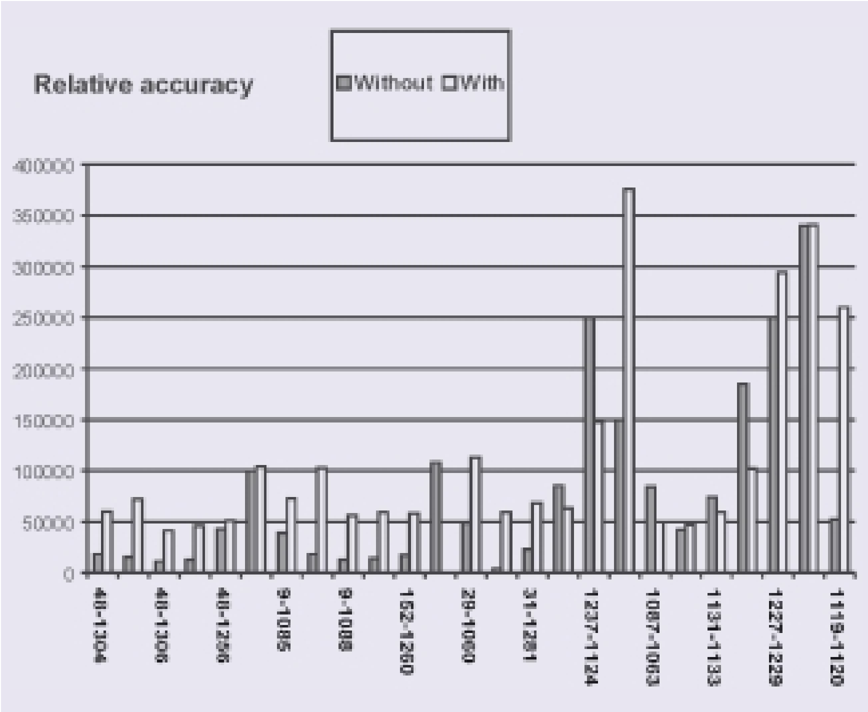


Fig. 3: Comparison of relative accuracies relying on the method of including

4 Comparing the results of including

The comparison of the tables 2 and 3 reveals that the differences of directly measured and computed lengths between the old trigonometric points and new points are remarkably reduced by using the model with preserved condition of the neighbourhood (Table 3) and that relative accuracy is considerably enlarged. The application of this model has removed the tensions in the network that has been made homogeneous (Fig. 3).

4.1 Analysis of some derived networks at the territory of the Republic of Croatia

On the basis of the data given by the State Geodetic Administration about some derived networks, i.e. "homogeneous field" the statistic analysis of the reliability of co-ordinates has been made.

According to the available data, the mean errors of all points registered in the State Geodetic Administration run (on the axes y and x) within the limits from 1.5 cm to 3.5 cm. For a part of these points it was possible to compute their standard deviations from GPS measurements and numeric processing. It has been found out that they run within the limits from 1.8 cm to 2.3 cm.

For the points determined by means of terrestrial method one can take the value of 2.5 cm for standard deviations S_y, i.e. S_x, and for those made by means of GPS measurements 2.0 cm. According to these data, the obtained standard deviations of the difference of co-ordinates obtained on the basis of terrestrial measurements and those obtained on the basis of GPS measurements runs up to, as related to the expression (FEIL, 1990), $s_{\Delta y}$ = 3.21 cm and $s_{\Delta x}$ = 3.21 cm.

Using these values the allowed differences of co-ordinates have been obtained in their absolute magnitude according to the expression (REICHARDT, 1995) that are presented in the Table 4.

$$|\Delta Y|_{allow.} = 1,96 \cdot 3,21 = 6,3 \text{ cm}$$

$$|\Delta X|_{allow.} = 1,96 \cdot 3,21 = 6,3 \text{ cm}$$

These are only approximate values of the allowed deviations for the entire network expressed in order to compare the results of including with or without preserving the neighbourhood. For the purpose of objective and thorough analysis one should express allowed deviations for each point obtained on the basis of their standard deviations.

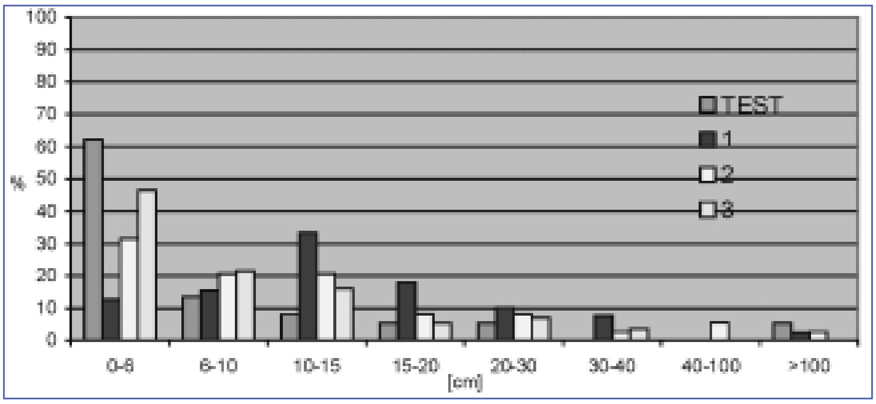


Fig. 4: Difference of co-ordinates obtained from terrestrial and GPS measurements

The Figure 4 shows, according to the previously mentioned criteria for |Δy| and |Δx|, that 62 % of the points have reliable co-ordinates, and that the differences have aroused in the results between the terrestrial and GPS measurements of random character. For 38 % of points the co-ordinate differences exceed the limits of the allowed deviation. In the last case, systematic errors are not excluded, or the aroused differences are the consequence of changing the position of single points. On the basis of this short analysis, presuming that the stated criterion for the allowed co-ordinate differences |Δy| and |Δx| of 6.3 cm is really acceptable, it can be concluded that in this network with the co-ordinates computed according to the model with preserved condition of the neighbourhood, 62 % of the points from the 2. to the 4. order can retain valid, old co-ordinates in the Gauss-Krüger co-ordinate system, and the other points should receive newly determined co-ordinates. For the sake of comparison, the analysis has been made in already mentioned networks from the territory of the Republic of Croatia where the model of including with the preserved condition of the neighbourhood has not been applied (Fig. 4).

That the suggested model is justifiable can be seen from Fig. 4. By applying the model of including with the preserved condition of the neighbourhood, 62 % of the points keep their old co-ordinates, and the including of GPS measurements into terrestrial systems where this model is not used, only 12.8 % to 46.4 % of the points in the network retain their old co-ordinates. The other points should receive new co-ordinates that would by all means make the maintenance of the existing cadastral plans very difficult.

5 Conclusion

By including GPS measurements into terrestrial systems according to the principal of preserving the neighbourhood we provide the increase of quality in the existing geodetic networks that is also homogenised at the same time, and the land related data that have been gathered for decades can be used efficiently furtheron. It is insignificant that also a larger number of old trigonometric points retains its co-ordinates in the state co-ordinate system. If this model would by any case not be used, great tensions, i.e. great mutual discrepancies, would arouse in the marginal areas of two neighbouring geodetic networks.

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Abstract

Poor preserved condition and unreliability of geodetic control points at the territory of the Republic of Croatia enforces the need for renewal. This paper presents the analysis of including satellite (GPS) observations into the state reference co-ordinate system. Two models of including are presented and analysed – a model with preserving the neighbourhood and without preserving the neighbourhood.