GPS Nonlinearly Precision Estimator and application

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On the base of nonlinear adjustment of GPS baseline vector, estimate value of unit weight mean square error, precision estimator of adjustment and baseline length are analyzed, and the simplified formulas for precision estimator of nonlinear adjustment of GPS baseline vector are put forward.

1 Estimate value of unit weight mean square error

According to literature [1], estimate value m_0 of unit weight mean square error can be solved by:

$$m_0 = \sqrt{\frac{V^T P V}{m-n-2}} \tag{1}$$

If the observations are equal weight and independence each other, p is the unit weight, the above formula is:

$$m_0 = \sqrt{\frac{V^T P V}{m-n-2}} = \sqrt{\frac{V^T V}{m-n-2}} \tag{2}$$

There, V in $V^T V$ can express by the correction of baseline vector X as follow:

$$V = A\,X + ({}^1/_2)X^T\,F\,X - W \tag{3}$$

In addition, we can transform formula (3) for:

$$V = \left(A, \frac{1}{2}F\right)\begin{pmatrix} X \\ Y \end{pmatrix} - W = B\Omega - W \tag{4}$$

There, $V = [V_1,\ V_2,\ \ldots V_m]^T$,

$$X = [\delta x_{ij}\quad \delta y_{ij}\quad \delta z_{ij}\quad V_{T1}\quad V_{T2}\ldots\quad V_{Tn}\quad \delta V_1\quad \delta N_2\quad \ldots\quad \delta N_k]^T,$$

$$Y = [\delta x^2\quad \delta x\delta y\quad \delta x\delta z\quad \delta y\delta x\quad \delta y^2\quad \delta y\delta z\quad \delta z\delta x\quad \delta z\delta y\quad \delta z^2]$$

$$W = [W_1\quad W_2\quad \ldots\quad W_m]^T,$$

$$b_{i1} = ({}^f/_c)DX^i_j,\ b_{i2} = ({}^f/_c)DY^i_j,\ b_{i3} = ({}^f/_c)DZ^i_j$$

$$A = \begin{bmatrix}
b_{11} & b_{12} & b_{13} & f & 0 & \ldots & 0 & -1 & 0 & \ldots & 0 \\
b_{21} & b_{22} & b_{23} & 0 & f & \ldots & 0 & -1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\
b_{n1} & b_{n2} & b_{n3} & 0 & 0 & \ldots & f & -1 & 0 & \ldots & 0 \\
b_{n+1,1} & b_{n+1,2} & b_{n+1,3} & f & 0 & \ldots & 0 & 0 & -1 & \ldots & 0 \\
b_{n+2,1} & b_{n+2,2} & b_{n+2,3} & 0 & f & \ldots & 0 & 0 & -1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\
b_{2n,1} & b_{2n,2} & b_{2n,3} & 0 & 0 & \ldots & f & 0 & -1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\
b_{m-n,1} & b_{m-n,2} & b_{m-n,3} & f & 0 & \ldots & 0 & 0 & 0 & \ldots & -1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \ldots & \vdots \\
b_{m-1,1} & b_{m-1,2} & b_{m-1,3} & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & -1 \\
b_{m,1} & b_{m,2} & b_{m,3} & 0 & 0 & \ldots & f & 0 & 0 & \ldots & -1
\end{bmatrix}$$

$$F = \begin{pmatrix}
a^1_{11}, a^1_{12}, \ldots, a^1_{1n}, a^1_{21}, \ldots a^1_{2n}, \ldots a^1_{nn} \\
a^2_{11}, a^2_{12}, \ldots a^2_{1n}, a^2_{21}, \ldots, a^2_{2n}, \ldots a^2_{nn} \\
\vdots \\
a^r_{11}, a^r_{12}, \ldots a^r_{1n}, a^r_{21}, \ldots a^r_{2n}, \ldots a^r_{nn}
\end{pmatrix}$$

There, m is the number of the error equation, n is the number of observe epoch.

2 Precision estimate of adjustment

Precision estimator of every sub-vector in unknown vector X is:

$$m_{x_i} = m_0\sqrt{\frac{1}{P_{x_i}}} \tag{5}$$

There, $i = 1, 2, \ldots, t$, $t = sv + 2$
sv is the number of satellite.
P_{x_i} is the weight of unknown x_i, it can be solved by the diagonal of N^{-1}:
Because,

$$\lambda = N^T M l,\ \tau = Q^T M l \tag{6}$$

According to the Cofactor Propagation Law, we can get:

$$Q_{\lambda\tau} = N^T M Q_{ll} M^T Q = N^T Q = 0 \tag{7}$$

That is, λ has no relative to τ. So we can get the cofactor matrix of the unknown [5]:

$$Q_{\hat{X}\hat{X}} = KK^T + KV_1K^T + \frac{1}{2}KV_pK^T \tag{8}$$

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where, $V_I = \sum_{i=1}^{n-t} I_i^2$, $V_P = (trP_iP_j)$, I_i is the *ith* plane of intrinsic curvature stereo matrix I_c, P_i is the *ith* plane of parametric effect curvature stereo matrix P_c.

$$N^{-1} = \begin{bmatrix} Q_{x_1x_1}, Q_{x_1x_2}, \ldots, Q_{x_1x_i} \\ Q_{x_2x_1}, Q_{x_2x_2}, \ldots, Q_{x_2x_i} \\ \cdots \\ Q_{x_ix_1}, Q_{x_ix_2}, \ldots, Q_{x_ix_i} \end{bmatrix}$$

Then,

$$P_{x_i} = \frac{1}{Q_{x_1x_1}} \tag{9}$$

So, we can get:

$$m_{x_i} = m_0\sqrt{Q_{x_ix_i}} \tag{10}$$

3 Precision estimate of baseline length b

According to literature [6], we can know,

$$b = \sqrt{(\Delta X_{ij}^0 + \delta x_{ij})^2 + (\Delta Y_{ij}^0 + \delta y_{ij})^2 + (\Delta Z_{ij}^0 + \delta z_{ij})^2} \tag{11}$$

Developing b on ΔX_{ij}^0, ΔY_{ij}^0, ΔZ_{ij}^0, then we can get:

$$b = b_0 + \frac{\Delta X_{ij}^0}{b_0}\delta x_{ij} + \frac{\Delta Y_{ij}^0}{b_0}\delta y_{ij} + \frac{\Delta Z_{ij}^0}{b_0}\delta z_{ij} +$$
$$(^1/_2)[\frac{b_0^2 - (\Delta X_{ij}^0)^2}{b_0^3}\delta x_{ij}^2 + \frac{b_0^2 - (\Delta Y_{ij}^0)^2}{b_0^3}\delta y_{ij}^2 +$$
$$\frac{b_0^2 - (\Delta Z_{ij}^0)^2}{b_0^3}\delta z_{ij}^2 - 2\times\frac{\Delta X_{ij}^0\Delta Y_{ij}^0}{b_0^3}\delta x_{ij}\delta y_{ij} -$$
$$2\times\frac{\Delta X_{ij}^0\Delta Z_{ij}^0}{b_0^3}\delta x_{ij}\delta z_{ij} - 2\times\frac{\Delta Y_{ij}^0\Delta Z_{ij}^0}{b_0^3}\delta y_{ij}\delta z_{ij} \tag{12}$$

There,

$$b_0 = \sqrt{(\Delta X_{ij}^0)^2 + (\Delta Y_{ij}^0)^2 + (\Delta Z_{ij}^0)^2} \tag{13}$$

And, $b = \sqrt{\Delta X_{ij}^2 + \Delta Y_{ij}^2 + \Delta Z_{ij}^2}$. Then, we can change above formula into:

$$\delta b = f^T X + (^1/_2)X^T F X \tag{14}$$

Formula (12) can express as:

$$b = b_0 + f^T X + (^1/_2)X^T F X \tag{15}$$

There,

$$f^T = \left(\frac{\Delta X_{ij}^0}{b_0}, \frac{\Delta Y_{ij}^0}{b_0}, \frac{\Delta Z_{ij}^0}{b_0}\right),\ X = (\delta x_{ij},\ \delta y_{ij},\ \delta z_{ij})^T$$

$$F = \begin{bmatrix} \frac{b_0^2-(\Delta X_{ij}^0)^2}{b_0^3}, \frac{\Delta X_{ij}^0\Delta Y_{ij}^0}{b_0^3}, \frac{\Delta X_{ij}^0\Delta Z_{ij}^0}{b_0^3} \\ \frac{\Delta Y_{ij}^0\Delta X_{ij}^0}{b_0^3}, \frac{b_0^2-(\Delta Y_{ij}^0)^2}{b_0^3}, \frac{\Delta Y_{ij}^0\Delta Z_{ij}^0}{b_0^3} \\ \frac{\Delta Z_{ij}^0\Delta X_{ij}^0}{b_0^3}, \frac{\Delta Z_{ij}^0\Delta Y_{ij}^0}{b_0^3}, \frac{b_0^2-(\Delta Z_{ij}^0)^2}{b_0^3} \end{bmatrix}$$

Assume:

$$X = \Delta + \mu_x \tag{16}$$

That is,

$$\Delta = X - \mu_x \tag{17}$$

There, $\mu_x = E(X)$, Then

$$E(\Delta) = 0,\ D_{\Delta\Delta} = D_{xx} = C \tag{18}$$

Join formula (17) into formula (15), we can get:

$$b = b_0 + f^T\Delta + f^T\mu_x + (^1/_2)\Delta^T F\Delta + \mu_x^T F\Delta + (^1/_2)\mu_x^T F\mu_x \tag{19}$$

Order:

$$Z = b - b_0 - f^T\mu_x - (^1/_2)\mu_x^T F\mu_x - (^1/_2)Tr(FC) \tag{20}$$

Then,

$$E(Z) = 0,\ D_{zz} = D_{bb} \tag{21}$$

From the above, we can get:

$$Z = f^T\Delta + (^1/_2)\Delta^T F\Delta + \mu_x^T F\Delta - (^1/_2)Tr(FC) =$$
$$(f + F\mu_x)^T\Delta + (^1/_2)Tr[F(\Delta\Delta^T - C)] = [f +$$
$$F\mu_x,\ F]^T[\Delta,\ (^1/_2)(\Delta\Delta^T - C)] \tag{22}$$

According to covariance error propagation law, we can get:

$$Q_{bb} = [f + F\mu_x,\ F]^T Q[f + F\mu_x,\ F] \tag{23}$$

There,

$$Q = m_0E\{[\Delta,\ (^1/_2)(\Delta\Delta^T - C)][\Delta,\ (^1/_2)(\Delta\Delta^T - C)]\} \tag{24}$$

In fact, μ_x is an unknown, we cannot solve it directly, we can solve μ_x by substituting $E(b)$ with X, thereof, we can get the estimator $\overline{Q_{bb}}$. *Tr* shows the trace of stereo array, trace of matrix is the sum of diagonal elements of matrix, it shows a value; trace of stereo array is the sum of diagonal elements of every face, it shows a vector. The computing method of trace of stereo array resembles that of matrix, it observes the character of matrix according to literature [2, 6].

Accordingly, estimation of mean square error of baseline length b is:

$$m_b = m_0\sqrt{Q_{bb}} \tag{25}$$

Estimation of relative mean square error of baseline length is:

$$m_r = \frac{m_b}{b}\cdot 10^6(ppm) \tag{26}$$

Now, let us show the importance and necessary of the development to the second terms and its intersection on the coordination and relative precision, compared to the classical method by example. There are three terms datum below, table 1 the raw GPS observations with 20-second interval, there are three terms observations, and table 2 is the corresponding weight of table 1.

(1). Raw data

Tab. 1: Raw GPS observations with 20-second interval

A			B				
1.276	0.363	0.458	0.190	– 0.190			
– 0.840	– 0.477	– 0.435		0.190	– 0.190		
0.535	– 0.020	0.388			0.190	– 0.190	
0.183	0.157	0.556				0.190	– 0.190
– 1.217	– 0.318	0.097					0.190
1.273	0.363	0.458	0.190	– 0.190			
– 0.836	– 0.480	– 0.429		0.190	– 0.190		
0.534	– 0.019	0.390			0.190	– 0.190	
0.183	0.157	0.547				0.190	– 0.190
– 1.218	– 0.319	0.098					0.190
1.269	0.363	0.455	0.190	– 0.190			
– 0.833	– 0.484	– 0.423		0.190	– 0.190		
0.532	– 0.017	0.392			0.190	– 0.190	
0.182	0.158	0.539				0.190	– 0.190
– 1.218	– 0.321	0.098					0.190

Tab. 2: The corresponding weight of table 1

0.417				
– 0.083	0.417			
– 0.083	– 0.083	0.417		
– 0.083	– 0.083	– 0.083	0.417	
– 0.083	– 0.083	– 0.083	– 0.083	0.417

Tab. 3:

p	0	0
0	p	0
0	0	p

Note: P in this table is the content of table 2.

(2). The results from classical method: table 4

Sequence number	Baseline	Distance	Azimuth	Δx	Δy	Relative precision
1	2000-1000	8985.9990	174 02′34.325″	– 0.0080	0.0015	1/1104011
2	2000-7001	2876.1230	219 58′28.874″	– 0.0025	0.0027	1/781624
3	2000-8001	3708.1902	229 52′10.399″	– 0.0020	0.0029	1/1052630
4	2000-1000	8985.9990	174 02′34.325″	0.0100	– 0.0052	1/797252
5	7001-8001	1004.8044	259 19′59.788″	0.0008	– 0.0009	1/834444
6	7001-1000	7284.8729	157 33′47.700″	– 0.0020	0.0015	1/2913949
7	8001-1000	7554.1563	150 04′52.079″	– 0.0027	0.0029	1/1906497

(3). Developing to the second and intersection: table 5

Sequence number	Baseline	Distance	Azimuth	Δx	Δy	Relative precision
1	2000-1000	8985.9990	174 02′34.325″	– 0.0070	0.0009	1/1277133
2	2000-7001	2876.1230	219 58′28.874″	– 0.0010	0.0007	1/2476253
3	2000-8001	3708.1902	229 52′10.399″	– 0.0018	0.0029	1/1079587
4	2000-1000	8985.9990	174 02′34.325″	0.0097	– 0.0046	1/836142
5	7001-8001	1004.8044	259 19′59.788″	0.0001	– 0.0006	1/1728856
6	7001-1000	7284.8729	157 33′47.700″	– 0.0010	0.0012	1/4524829
7	8001-1000	7554.1563	150 04′52.079″	– 0.0017	0.0024	1/2593377

4 Conclusion

On the base of traditional precision estimator method of GPS baseline calculation with linear, nonlinear function of mean square error estimator of unit weight, precision estimates of adjustment and of GPS baseline are developed to the second-order terms and the intersection-order terms with Taylor's series in this paper. The operation of stereo array and the trace of stereo array are come down, There is no ideal formula for nonlinear precision estimator of GPS baseline calculation; that is to say, it is difficult to estimate precision. In allusion to the above difficulties, the simplified formulas for precision estimate of GPS baseline vector nonlinear adjustment are put forward in this paper.

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Abstract

On the base of nonlinear adjustment of GPS baseline vector, estimate value of unit weight mean square error, precision estimator of adjustment and baseline length are analyzed, and the simplified formulas for precision estimator of nonlinear adjustment of GPS baseline vector are put forward in this paper.

In the practice and application of surveying and mapping, often meet many mathematical questions about nonlinearly. Such as many designed models and quality rules in optimize design of net adjustment are nonlinear; there is question of nonlinear function in model of adjustment; in data disposal of deformation observation, only nonlinear model can reflect the impersonal deformation law of plasmodium better. During precision estimator of GPS baseline nonlinear calculation, the traditional method is to do by iterative method to meet the precision's need. But the data shows that iterative method is complicated and difficult. By linear method, developing the nonlinear function of GPS baseline vector to the first-order terms with Taylor's series. Of course, if the approximation of the unknown we choose is closer to the true value; the correction of observation is little, and the no linearity is weaker, it can be accepted to develop it with Taylor's series lose sight of the second-order terms and the others. The result can meet the precision's need. Other wise, it cannot meet the precision's need.

Base on nonlinear adjustment of GPS baseline vector, precision estimator method of GPS baseline nonlinear calculation is discussed and put forward according to nonlinear adjustment theory. Simplified formulas of estimate value of unit weight mean square error, precision estimates of adjustment and baseline length are given in this paper.

Key word: Nonlinear Adjustment, Precision Estimator, GPS Positioning, Baseline Vector