Analysis of Errors in Distance Measurement with measuring tape

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Verschiedene Parameter beeinflussen die Präzision der Bandmaßmessung. Die Einflüsse dieser Parameter werden untersucht und mit Beispielen unterlegt.

1 Introduction and research goal

The purpose of every measuring is determination of measuring quantity which is expressed in measuring result gained during expertly and conscientiously conducted measuring procedure in which adequate measuring instruments and tools are used. The expressed value of the measuring result should come close enough to the actual value. Precisely how close it should be depends on installed criteria which can be prescribed by the code book, determined by the investor, or the highest possible accuracy to be reached in given circumstances is required.

In every process of measuring there are imperfections which can cause an error in the measuring result, but since the actual value itself is an idealized concept so the error is an idealized and undetermined quantity. Since the criteria are often placed really high, and working conditions are not ideal, the role of a geodetic expert in such circumstances is highly demanding in respect of the selection of proper instruments and equipment and method of measuring in order to meet the requirements.

Such very high criteria in view of accuracy were placed with installation of factory machines in the Factory of Light Metals (TLM) in Šibenik. Required accuracy during measuring of distance from 3,0 m to 120,0 m was 0,05 mm/1 m, and temperature during the work ranged from + 6 °C to + 40 °C.

Since the distance were short, and the required accuracy was very high, the question is which method and which measuring equipment can satisfy the requested accuracy.

2 Distance measurement

Lengths are nowadays measured in various ways. The oldest, and still often used method, is direct distance measurement (Solarić and Bilajbegović 1992) with invar and steel tapes.

Electronic distance meters, which can vary in construction and accuracy, are more and more often used. However, in the area of a factory, where there are influences of refraction, reflection, vibration and turbulence of air layers and movement of stands because of the operation of machines, not even most precise distance meters could meet the required criteria.

Due to bad working conditions in the field, the conclusion is that the requirement can only be fulfilled by measuring the distances with invar or steel tapes.

2.1 Analysis of errors

Measuring serves to obtain information on values of measured quantities with pre-defined accuracy. On the basis of how all errors influence the result of measuring, a choice of adequate instruments, working method and the most suitable time of work can be made. For the purpose of objective estimation of measuring accuracy it is of high importance to determine the value and the nature of effect of errors, especially the significant ones, because by the reduction of their effect the accuracy of measuring can be increased.

Classification of errors according to the type of effect could be carried in the following way (Mihailović and Vračarić 1985):

- *Error in setting the starting point of the tape to the starting point of the distance measured*, as wess as the error in reading at the end point of the distance is accidental and it can be reduced by a working method
- Error in length due to the *wrongly determined difference in height* is also accidental, and it is of a small amount since the distances are mostly short
- Unlike the previous two, *an error in comparing the tape to the result of measuring* acts like systematic error of an unknown sigh and size and it is therefore necessary to deal with it in detail
- *Error in length caused by the difference in temperature during measuring and the one during comparing* can be accidental or systematic. It is systematic if measuring is carried out at the temperature different than the temperature during comparing the tape. This was the case in the Factory of Light Metals in Šibenik
- Likewise, the *error caused by inconsistent pull* can be accidental or systematic

Everything stated above clearly shows that errors connected with the tape comparison, force of protraction and the change of temperature could amply influence the required accuracy. Therefore, special attention is paid to that problem.

3 Review of the strained tape theory and the impact of parameters

Strained measuring tape for distance measurement behaves in accordance with the theory of catenary whose distance between the end points is not constant but it depends on several parameters. The parameters are the force of expansion, sag and the change of temperature. By applying the theory of catenary with constant weight per unit of length, we will carry out the analysis of impact of the parameters to the change of compared tape length.
On the basis of Fig. 1 there is

$$\mathrm{tg}\alpha = \frac{q \cdot x}{F_H} \text{ i } tg\alpha = \frac{d\,y}{d\,x} \tag{1}$$

where: q – weight of the tape per one meter
F – horizontal force straining the tape
h – sag of the tape
L – length of the tape's arc

By equation both expressions we get:

$$\frac{d\,y}{d\,x} = \frac{q \cdot x}{F_H} \tag{2}$$

Equation (2) presents a differential equation of parabolic catenary.
By integration of equation above we get:

$$y = \frac{q \cdot x^2}{2F_H} + C \tag{3}$$

From the edge properties x = 0, y = 0 follows C = 0 and therefore:

$$y = \frac{q \cdot x^2}{2F_H} \tag{4}$$

For $x = l/2$ and y = h equation (4) becomes:

$$y = h = \frac{ql^2}{8F_H} \quad \text{or} \quad F_H = \frac{ql^2}{8h} \tag{5}$$

The length of the tape's curve element can be expressed as:

$$d\,L = \sqrt{d\,x^2 + d\,y^2} = \left(\sqrt{1 + \left(\frac{d\,y}{d\,x}\right)^2}\right) d\,x \tag{6}$$

or after the development into binominal line we get:

$$\mathrm{dL} = \left[1 + \frac{1}{2}\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2 + \ldots + \right] \mathrm{dx} \tag{7}$$

On the basis of (2), the equation (7) is as follows:

$$dL = \left[1 + \frac{1}{2} \cdot \frac{64h^2}{l^4} x^2 + \ldots\right] dx \tag{8}$$

Total length of the suspended tape is:

$$L = 2\int_0^{1/2} \left[1 + \frac{1}{2} \cdot \frac{64h^2}{l^4} x^2 + \ldots\right] dx = l + \frac{8}{3} \cdot \frac{h^2}{l} \tag{9}$$

In the case of measuring tapes, L is compared length of the tape and l is the length measured. Expression (9) is multiplied by l and we get:

$$l^2 - Ll - \frac{8}{3}h^2 = 0 \tag{10}$$

Real solution of the square equation above is in the following form:

$$l = \frac{1}{2}\left(L + \sqrt{L^2 - \frac{32}{3}h^2}\right) \tag{11}$$

In the equation above, h is measured directly of calculated from the equation (5) and l is determined as decrease of compared tape's length L because of the sag h. Error in distance measurement caused by sag can be expressed as:

$$\Delta L(h) = l - L \tag{12}$$

Similarly, tape stretched by the horizontal force (F = 50 or F = 100 N) also changes the compared initial length according to Hooke's law:

$$\Delta L(F_H) = \frac{F_H \cdot l}{E \cdot A} \tag{13}$$

where: E – elasticity module of the tape and A – area of tape's cross section.
Finally, the change in compared length of the tape is caused by the change of the temperature in relation to the temperature at time of comparing the tape. The change is described in the following expression:

$$\Delta L(t) = \alpha_t l(t - t_0) \tag{14}$$

where t – temperature during measuring, and t_0 – declared temperature during tape comparing.

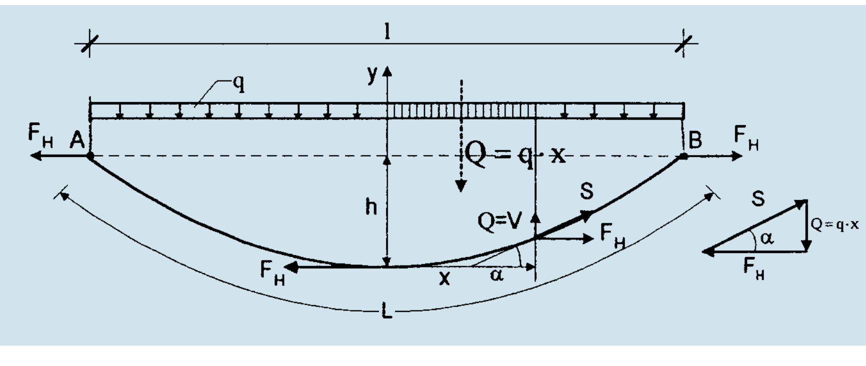

Fig. 1: Strained tape

Total change in the length of the tape is a consequence of all possible changes described above and it comes to:

$$\Delta L = \Delta L(F_H) + \Delta L(h) + \Delta L(t) \quad (15)$$

4 Experimental determination of relevant parameters

As it can be seen from the expressions (11), (13), (14) and (15), in order to determine total change of tape's length, it is necessary to know the sag of the tape h at certain horizontal force F, and elasticity module of material E the tape is made of, as well as the temperature coefficient of linear elongation α_t. All those parameters (h, E and α_t) must be determined experimentally.
That is why we have approached experimental determination of those parameters. On the basis of experimental measuring it was determined that theoretical value of the sag of the tape h is equal to the measured one. On the other hand, values of elasticity module E and temperature coefficient α vary significantly and often are not in accordance with the usually used values (for steel: $E = 2 \cdot 10^5$ MPa and $\alpha_t = 1{,}2 \cdot 10^{-5}/\mathrm{K}$).
In order to precisely determine total change in the length of the tape, and thereby accurately estimate and cancel errors in distance measurement, it is necessary to check elasticity module of each tape as well as their thermal coefficient of expansion per unit of temperature difference. Experimental analysis of the two parameters for four types of tapes (two steel and two invar tapes) is presented in this paper.

4.1 Determination of elasticity modules of measuring tapes

Experimental determination of elasticity modules of tapes was carried out on pieces of tapes of certain length which were taken as samples of actual tapes. The pieces of tapes were c/a 200 mm in length, and measuring tapes for measuring deformation (tensiometers) were attached to their middle on bith sides. The pieces of tapes (samples) were perforated in the axis and loaded with longitudinal axial force in the area of elasticity. During that, relative deformations were measured using electric-resistant tapes and Wheatson's bridge. By means of the diagram in Fig. 2, elasticity module of each tape was determined.
The values of elasticity modules for different tapes made from different materials, gained by the procedure above, are shown in Table 1.
The table above shows that elasticity modules of invar and steel tapes are different. They also differ within invar tapes, and all of them differ from usually used values of elasticity modules of tapes ($E = 2 \cdot 10^5$ MPa).

4.2 Determination of thermal coefficient of linear expansion

Experimental determination of thermal coefficient of linear expansion was carried out on samples of tapes in a special room where the temperature of tape samples was gradually increased. For every registered increase in temperature of each sample, a measuring of length was executed. In that way, we were able to determine linear expansion of each sample for temperature increase

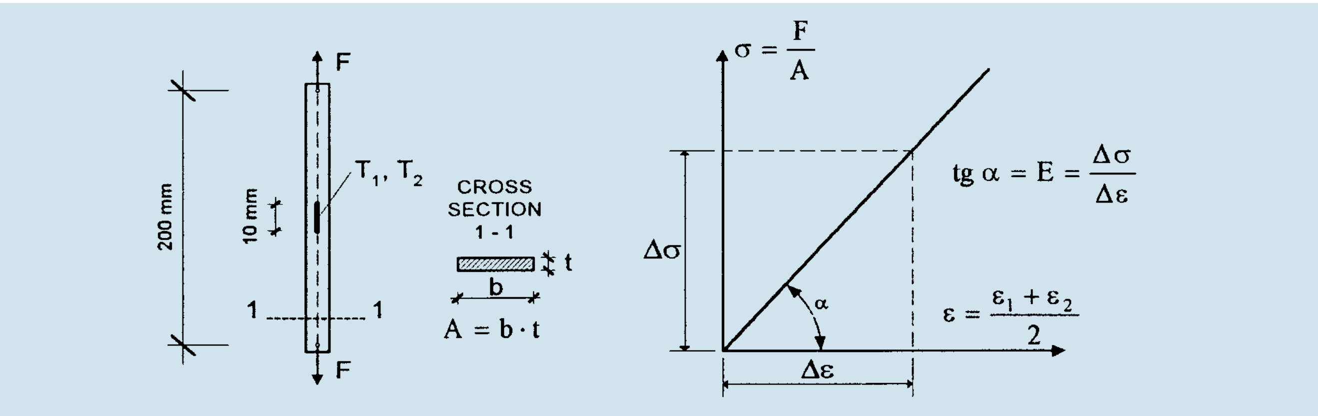

Fig. 2: Principle of measuring elasticity modules of tape

Tab. 1: Elasticity modules of tapes determined by measuring

Tape	*b (mm)*	*t (mm)*	*$A = b \cdot t$ (mm^2)*	*E (MPa)*
Steel	12,89	0,18	2,32	$2{,}33 \cdot 10^5$
Steel plastic-coated	13,01	0,16	2,082	$2{,}31 \cdot 10^5$
Invar 1	10,08	0,31	3,125	$1{,}93 \cdot 10^5$
Invar 2	5,95	0,46	2,74	$1{,}61 \cdot 10^5$

Table 2: Measured thermal coefficient of expansion α_t

Tape	*Thermal coefficient α_t (1/K)*
Steel	$1{,}558 \cdot 10^{-5}$
Invar 2	$0{,}0035 \cdot 10^{-5}$

per unit. Results of measuring thermal coefficients of expansion of tapes (invar and steel tapes) are shown in Table 2.

The results above show that invar tape hardly reacts to the change of temperature, so the length change due to temperature can be ignored with invar tapes, but other parameters remain present. With steel tapes, temperature coefficient α_t is considerable and it is larger than the one usually used ($\alpha_t = 1{,}2 \cdot 10^5/\mathrm{K}$), so it should be examined experimentally.

5. Numerical examples of calculation of tape's total length

A numerical analysis of steel tape's length change due to sag, expansion and temperature change was carried out. Two standard expansion forces $F_{H1} = 50\ N$ and $F_{H2} = 100\ N$ were analyzed.

Total change of the compared tape's length is manifested as measuring error. It can be noticed that changes in primary compared length are considerable and that they should be taken into consideration during measuring, especially with longer steel tapes. Naturally, the same theory can be applied to invar tapes and invar wires, excent that here the effect of temperature is negligible, and they are shorter so the effects of expansion and sag are smaller. Results of analysis of changeable compared length of steel tapes of giben characteristics are shown in table 3 and 4.

In Figure 3 the analysis is presented graphically. Diagrams show the change of length of plastic-coated steel

Table 3: Alteration of length of steel measuring tape in mm at the expansion force $F_{H1} = 50\ N$

Initial length of the tape L (m)	*Size of the sag h_1 (mm)*	*Change of length due to the sag l_1 (mm)*	*Change of length due to F_{H1} $\Delta L\ (F_{H1})$ (mm)*	*Change of length due to h_1 $\Delta L(h_1)$ (mm)*	*Change of length due to $t - t_0 = +1\ K$ $\Delta L(t - t_0 = +1\ K)$ (mm)*	*Full change of length $\Delta L = \Delta L(F_{H1}) + \Delta L(h_1) + \Delta L(t - t_0 = +1\ K)$ (mm)*
5,0	12,406	4999,918	+ 0,519	– 0,082	+ 0,078	+ 0,515
10,0	49,625	9999,343	+ 1,039	– 0,657	+ 0,156	+ 0,538
	(48,70)*					
15,0	111,656	14997,783	+ 1,559	– 2,216	+ 0,234	– 0,423
20,0	198,50	19994,745	+ 2,078	– 5,255	+ 0,312	– 2,865
	(189,70)*					
25,0	310,156	24989,735	+ 2,599	– 10,265	+ 0,390	– 7,276

* – measured size of the sag

t – temperature during measuring

t_0 – temperature during comparison of the tape

$E = 2{,}31 \cdot 10^5$ Mpa, $A = 2{,}082\ \mathrm{mm}^2$, $q = 0{,}1985 \cdot 10^{-3}\ \mathrm{N/mm}'$

Table 4: Alteration of length of steel measuring tape in mm at the expansion force $F_{H2} = 100\ N$

Initial length of the tape L (m)	*Size of the sag h_1 (mm)*	*Change of length due to the sag l_1 (mm)*	*Change of length due to F_{H1} $\Delta L\ (F_{H1})$ (mm)*	*Change of length due to h_1 $\Delta L(h_1)$ (mm)*	*Change of length due to $t - t_0 = +1\ K$ $\Delta L(t - t_0 = +1\ K)$ (mm)*	*Full change of length $\Delta L = \Delta L(F_{H1}) + \Delta L(h_1) + \Delta L(t - t_0 = +1\ K)$ (mm)*
5,0	6,203	4999,979	+ 1,039	– 0,020	+ 0,078	+ 1,097
10,0	24,812	9999,836	+ 2,079	– 0,164	+ 0,156	+ 2,071
	(24,70)*					
15,0	55,828	14997,446	+ 3,118	– 0,554	+ 0,234	+ 2,798
20,0	99,250	19998,686	+ 4,158	– 1,313	+ 0,312	+ 3,157
	(97,70)*					
25,0	155,087	24997,434	+ 5,1999	– 2,565	+ 0,390	– 3,024

* – measured size of the sag

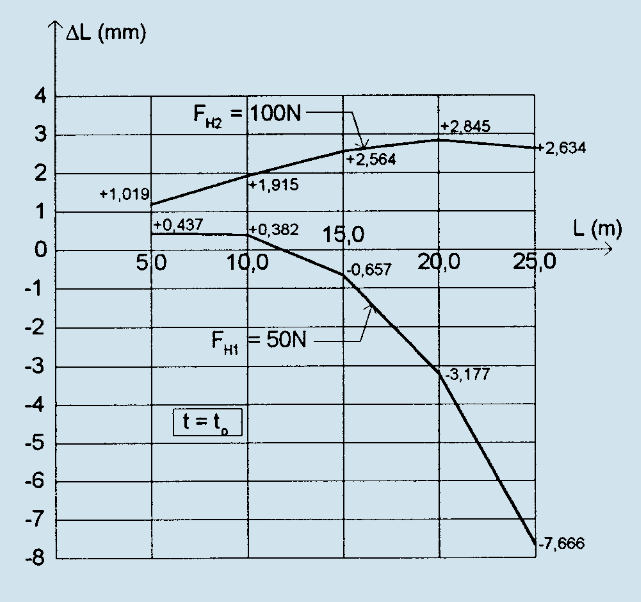

Fig. 3: Graph showing total change of tape's length at the temperature of comparing

measuring tape caused by the impact of the force magnituce F and sag h $(\Delta L(F_{H1}) + \Delta L(h_1)))$ at the temperature of comparing $(t = t_0)$, that is without the effect of temperature.

6 Conclusion

One of the basic measuring dimensions determined in geodesy are distances. Electronic distance meters are nowadays most often used for their determination.
However, because of the conditions in the field and required accuracy with short distances, on the basis of thorough analysis of error sources, it was concluded that distances should be measured with invar and steel tapes. After measuring with that equipment, some aberrations appeared. Therefore, we decided to examine the effect of systematic errors, especially of those connected to the comparison of tapes, because those errors are most important.
We analyzed the impacts of elasticity modules of measuring tapes (E), thermal coefficient of expansion (α_t, sag (h) and the expansion force (F) to the accuracy of measuring.
With steel tapes (regular or plastic-coated) mutual differences in elasticity module and thermal coefficient of expansion were determined, other than those usually used with steel.
With invar tapes this refers only to the elasticity module, while the thermal coefficient is in accordance with the expected theoretical value.
The conclusion is, therefore, that during every precise measuring with measuring tapes a potential error that can occur should be borne in mind.
That is why an elasticity module and a thermal coefficient of expansion should be experimentally determined if the steel tapes are used, and for the invar tapes only elasticity module, and all that should be taken into consideration during the analysis of measuring results.
Apart from the effects mentioned, alternation caused by sag plays an important role in the accuracy of measurement and therefore it should be taken into account, depending upon the force of protraction.

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Summary

Errors made during precise distance measurement with measuring tape are analyzed in this paper. The distance between end points of measuring tape depends upon several parameters: force of expansion, sag, change of temperature, length of the tape, material the tape is made of and weight. In order for those parameters to be taken into account, it is necessary to determine mechanical characteristics of the material the tape is made of.

The paper presents the numerical example of how those parameters influence the accuracy of measuring with steel plastic-coated tape of various lengths and two magnitudes of force of expansion.