Empirical Investigation of a Stochastic Model Based on Intensity Values for Terrestrial Laser Scanning

Empirische Untersuchung eines auf Intensitätswerten basierten stochastischen Modells für terrestrisches Laserscanning

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Stochastic information of measurements has always been an important part in the field of geodesy. Especially for engineering tasks like deformation monitoring the knowledge of achieved precision is of vital importance. For more than a decade terrestrial laser scanners have been used for high precise measurement applications, however there has been a lack of focus in the literature of an adequate stochastic model for terrestrial laser scanning. This contribution presents a comprehensive empirical experiment containing numerous laser scans at various scan configurations in order to examine a stochastic model based on intensity values. The results confirm the fundamental suitability of intensity values for a stochastic modelling but also point out certain problems concerning scanning geometry. High incidence angles in laser scanning can lead to an improvement of the precision in object's surface normal direction, which agrees with a previously proposed theoretical model for positional uncertainty of laser scan 3D points and should not be ignored in further development of a stochastic model based on intensity values. Based on these results a stochastic model was estimated and the precision of 3D points explicitly expressed as function of intensity and incidence angle.

Keywords: Intensity, stochastic modelling, scan configuration, terrestrial laser scanner

Stochastische Informationen sind ein wichtiger Bestandteil von geodätischen Messungen im Bereich der angewandten Geodäsie. Speziell für Ingenieuraufgaben, wie beispielsweise Deformationsmessungen, ist die Kenntnis der erreichten Präzision essenziell. Seit mehr als einem Jahrzehnt werden terrestrische Laserscanner für hochpräzise Aufgaben eingesetzt, jedoch wurde die Entwicklung eines geeigneten stochastischen Modells für terrestrisches Laserscanning bisher vernachlässigt. Dieser Beitrag zeigt eine umfassende empirische Untersuchung von 3D-Laserscanning unter verschiedenen Aufnahmekonfigurationen. Dazu soll ein stochastisches Modell basierend auf Intensitätswerten näher untersucht werden. Die Ergebnisse bestätigen die grundsätzliche Eignung von Intensitätswerten für die stochastische Modellierung, zeigen aber auch Probleme bezüglich der Scankonfiguration auf. Flache Auftreffwinkel können zu einer Verbesserung der Präzision in Richtung der Objektnormalen führen. Dies wird von einem bereits vorgeschlagenen theoretischen Modell für die Punktunsicherheit bei 3D-Laserscans bestätigt und sollte in der weiteren Entwicklung stochastischer Modellierung für terrestrisches Laserscanning nicht ignoriert werden. Aufbauend auf den Ergebnissen wird ein stochastisches Modell geschätzt und die Präzision von 3D-Punkten als Funktion von Intensitätswert und Auftreffwinkel explizit angegeben.

Schlüsselwörter: *Intensitätswerte, stochastische Modellierung, Aufnahmekonfiguration, terrestrische Laserscanner*

1 INTRODUCTION

Nowadays, the technology of Terrestrial Laser Scanning (TLS) is used more frequently for a wide field of applications such as deformation monitoring /Mechelke et al. 2012/, cultural heritage /Lichti & Gordon 2004/ or reverse engineering. Due to its ability of measuring dense 3D point clouds TLS has become increasingly popular for scanning complex objects with high demands in terms of accuracy and precision. Hereby, precision is defined as agreement of measurements, i. e. scattering of values, measured within equal conditions and accuracy as the closeness to a true value /ISO 17123-1/. Stochastic information of the observations is essential for a rigorous evaluation of the achieved results, to identify outliers and to allow statistical testing, i. e. to test the significance of the deformation and has become more popular in recent publications e. g. /Neuner et al. 2016/, /Wunderlich et al. 2016/. These values should be also considered in the pre-planning of deformation applications, for instance, as a criterion for view point planning. The precision of points scanned from a certain view point needs to be known when finding the optimal solution that satisfies the demands in quality of a point cloud by minimal economical effort, e. g. number or resolution of scans /Wujanz et al. 2016a/. This becomes more sophisticated when deformation analysis is performed by TLS devices and hence a specific precision in a certain object direction is required /Wunderlich et al. 2016/.

Furthermore, the prior knowledge of stochastic information for weighting is desirable for adjustment procedures as stated in /Ghilani 2006/: "The weight of an observation is a measure of its relative worth compared to other measurements. Weights are used to control the sizes of corrections applied to measurements in an adjustment." Nevertheless, stochastic information were often neglected in the past and 3D point clouds are usually weighted equally which is not adequate as shown in several publications, e. g. /Böhler et al. 2003/. The precision of each single 3D point differs and should be taken into account when establishing a stochastic model. /Soudarissanane et al. 2011/ categorises the factors influencing point quality during the scanning procedure into four main groups, namely:

- Scanner mechanisms /Neitzel 2006/,
- Scanning geometry /Soudarissanane et al. 2011/, /Kersten et al. 2008/,
- Atmospheric conditions and environment /Pfeifer et al. 2007/, and
- Object properties /Pesci & Teza 2008/.

Former studies derived stochastic information by post data capturing e. g. by scanning geometric primitives and by computing the deviations between the measured point cloud and the target geometry, such as the example in /Alkan & Karsidag 2012/. Another method is to use reference observations e. g. from a total station, for determining this information /Zámečníková et al. 2015/. /Elkhrachy & Niemeier 2006/ presented three simple approaches in which a stochastic model is derived, e. g. by using the slope distance to an object. Additionally, calculating the theoretical positional uncertainty based on the distance and angular uncertainties provided by manufactures is possible. For instance, /Bae et al. 2007/ created a closed-form expression for 3D points. Nevertheless, the theoretical uncertainty does not always coincide with actual achieved uncertainty since influences as object properties are not considered. As /Bae et al. 2007/ used the term positional uncertainty to describe the calculated theoretical uncertainty of a single 3D point, this term will be adopted when discussing this contribution in Section 4. /Gordon 2008/ introduced a more comprehensive model where systematic and random errors of internal as well as external sources are considered. The problem with this is that it requires the exact knowledge of the previously mentioned error groups and their effects for respective objects. Since such details are rarely known or obtainable for each single point a less complex model is desirable. However, regarding the fact that the precision of scan points mainly depends on the reflectorless distance measurement – the elementary observation in TLS – the priority for stochastic modelling should be focused here.

In /Wujanz et al. 2016b/ a telling stochastic model is established. Therein stochastic information is calculated based on intensity values, which are usually recorded and describe the amount of the reflected signal strength. It was shown that a strong relationship between the received signal strength and the scanning noise exists. Almost the same influences which decrease the precision of points also decrease the signal strength, mainly distance, incidence angle and surface properties for a measured point cloud. Hence, the intensity values can be evaluated as adequate indicator for stochastic information. /Wujanz et al. 2017/ confirm this statement in several empirical experiments and describe the a-priori variances for a single point explicitly as an exponential function of intensity. The advantages for stochastic modelling based on intensity are summarised as follows:

- The Precision of range measurements is easily calculable as function of intensity,
- No knowledge about surface properties as well as scan configuration is required,
- Individual stochastic information for each resulting 3D point, and
- No limitation due to data volume or object complexity.

Former experiments of the previous publications focused on the precision in 1D-scanning mode which means the precision of distance measurements in laser beam direction. Furthermore, the influence of the incidence angle onto the precision in laser beam direction is investigated in several contributions e. g. /Wujanz et al. 2017/. Our article investigates in the precision of distance measurements not only in laser beam direction but in Euclidean space. It will be shown that the impact of the incidence angle onto the precision in laser beam direction is different to the impact of the incidence angle onto the precision in any other direction, e. g. in surface normal direction. This variable impact cannot be covered and represented solely by the intensity value.

Processing software for point clouds often provide scaled intensity values but for a meaningful stochastic model, raw intensity values are required. The applied Leica C10 laser scanners provide raw intensity values in 212 increments (inc), meaning the measurable values range between –2 048 to 2 047 inc. It should be emphasised that this contribution focuses solely on the resulting precision of scan points provoked by scan configuration and object's surface properties. Systematic errors and overall accuracy are not considered.

The paper is structured as follows: In Section 2 the data acquisition is described which forms the basis for further investigations in Section 3. The findings are evaluated in Section 4, a functional model is estimated in Section 5 and discussed in Section 6. A conclusion and outlook is given in Section 7.

2 DATA ACQUISITION AND PREPARATION

The data capturing was performed by using three different Leica C10 laser scanners in order to demonstrate the variability between scanners of the same model. The target in *Fig. 1* consists of a grid of 20 evenly distributed grey scaled segments ranging from white to black with a size of 10.5 cm × 11.5 cm and was scanned at different distances from 1 m to 50 m at 1 m (and shorter) intervals. In addition, the target was also captured by one of the scanners at approximately regular intervals of incidence angles of 5° between 0° to 80° (*Fig. 2*). We define the incidence angle as angle between surface normal and transmitted laser beam direction. In this contribution only the data from latter mentioned scanner will be presented since all scanners show similar trends in the results. The point spacing was set at 1 point per mm^2 at respective distance for perpendicular scans. A measurement tape and a compass were used for the target set-up. Note that a high absolute accuracy was not required since this experiment focuses on relation between intensity and precision. Also note the smoothness of the target is far below the expected laser scanner's precision. It should be mentioned that the involved laser scanner has been verified on a regular basis based on a self-calibration method provided by /Lichti & Licht 2006/ and is free of significant systematic influences. Apart from that, small biases and systematic errors would not have a significant influence on the resulting precision.

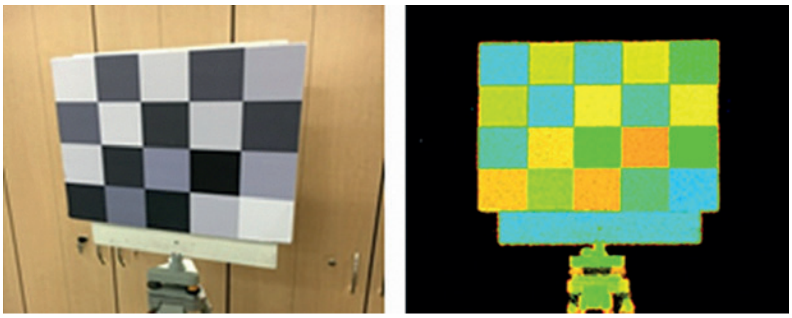

Fig. 1 | Target used for scanning (left) and point cloud of single scan (right)

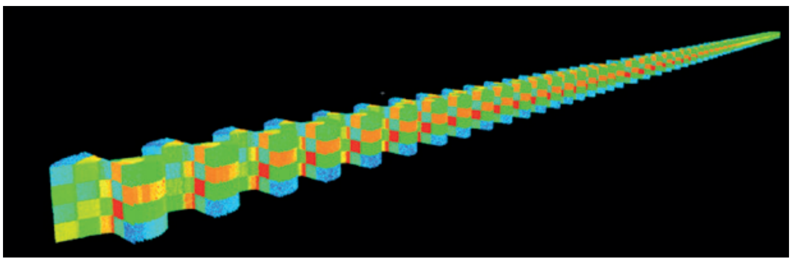

Fig. 2 | Combined point cloud of all target scans

After data capturing, more than a thousand individual scans at multiple distances and incidence angles were available for processing. This requires the various grey scaled regions for the target to be extracted and the points for each of the gridded areas to be segmented. In this instance, the planar segments were automatically isolated and extracted using a direct search optimisation method /Dennis & Torczon 1991/. The individual grey shaded segments were extracted by transforming the data to fit the grid pattern of the target such in order to minimise

$$\operatorname{argmin}\left(\sum_{j=1}^{20}\sum_{k=1}^{n} x_{j,k}\left(I_{j,k}-\overline{I}_j\right)\right), \qquad (1)$$

where $I_{j,k}$ is the intensity value for point k, $\overline{I}_j$ is the mean intensity value for the points in segment j, and $x_{j,k}$ is defined as 1 if point k belongs to segment j, and 0 otherwise as in Eq. (*2*):

$$x_{j,k}=\begin{cases}1 \text{ if } x_{j,k} \in \text{grid cell } j,\\ 0 \text{ if } x_{j,k} \notin \text{grid cell } j.\end{cases} \qquad (2)$$

Because this is essentially a binary integer programming problem, the simple direct search method was applied to solve the transformation parameters of the grid model to the points, which minimise the difference between the intensity value of a point and the mean intensity value of the grid segment to which the point belongs. Other methods such as clustering and region growing of the intensity values may also be used. However, because some of the intensity values between segments were less than the variation of the intensity values within the segments and because the target edition was known, the above approach was chosen. In less than 1 % of scans the procedure failed; those segments were excluded from further processing. The points of each target were first transformed onto the plane of best fit using Principal Component Analysis (PCA) for the transformation parameter estimation. PCA uses the covariance-structure of the data and yields to a covariance matrix including the eigenvectors indicating the surface normal direction of a target /Johnson & Wichern 2002/. Since the blooming effect between different segments caused by the effect of mixed pixels was present in the data, a buffer of ±5 mm to either side of the edges were subsequently removed based on the nominal size of the laser beam. For the plane estimation PCA was used including a robust data snooping according to /Leys et al. 2013/.

The segments tend to have an offset to the overall target plane since various colours lead to systematic errors in range due to their reflectivity /Pesci & Teza 2008/. In our experiment the distance measurements of dark segments are slightly elongated while the segments close to a white colour are shortened up to 2 mm. Since this shift does not affect the precision directly, this matter is only mentioned.

Fig. 3 shows the resulting point cloud of the target coloured by the intensity values (top) and the intensity residuals to mean intensity value of respective segment (bottom). Overall the covered intensity spectrum was –1 800 to 200 inc, while the intensity interval from white to black segments within a target reaches about 800 inc. The Root Mean Square (RMS) of intensity within a single segment is around 30 inc and darker segments are in general less noisy. Note that the relation between colour reflectance and intensity increments is non-linear. Increment steps are smaller for higher intensity values, hence the variation particularly residuals of intensity increase for lighter segments.

After segmentation descriptive statistics for each of the more than 20 000 segments j on the target t, as well as for the overall target could be calculated. These included the

- mean distance ($d_{t,j}$),
- mean incident angle ($\theta_{t,j}$),
- mean intensity value ($\overline{I}_{t,j}$),
- RMS of intensity within one segment ($RMS_{I_{t,j}}$),
- RMS to the fitted plane ($RMS_{t,j}$),
- average offset of the segment to the target ($o_{t,j}$), and
- the number of points ($n_{t,j}$).

Hereby the $RMS_{t,j}$ of fitted plane represents the noise level in segment normal direction as calculated in Eq. (*3*) using the orthogonal distances res_k of n points k to corresponding estimated plane:

$$RMS_{t,j} = \sqrt{\frac{\sum_{k=1}^{n}(res_k)^2}{n}}. \qquad (3)$$

3 RELATION BETWEEN INTENSITY, PRECISION AND SCAN CONFIGURATION

Starting with the main aspect, namely the relationship between intensity and precision, *Fig. 4* presents the RMS to the fitted plane of all segments and the mean intensity value of respective segments. The colour as shown in the colour map indicates the incidence angle. A coherent distribution is clearly visible: segments of each incidence angle follow a specific curve shape. A similar curve progression for perpendicular scans (here blue points) is also found in /Wujanz et al. 2017/ and confirms the theory that precision is a function of intensity. Nevertheless, a correlation between incidence angle and RMS can be obtained. Higher incidence angles lead to a decrease in RMS values even though to a decrease in captured intensity.

In order to investigate further into the relationships, the data is plotted for all incidence angles and distances with their mean intensity value for each segment (*Fig. 5*). The colour presents the RMS of the best plane fit for each segment. The intensity values evenly decrease for increasing distances as well as incidence angles. For distances less than 20 m with intensity values above 1 500 inc the RMS hardly exceeds 2 mm. It is conspicuous in having the best results for RMS at high incidence angles since these angles are generally undesired and well known as "bad" scanning geometry leading to elongated footprints of the laser beam. /Soudarissanane et al. 2011/ stated in this content that higher incidence angles lead to more noise in the scans and provoke an unfavourable Signal-to-Noise Ratio (SNR).

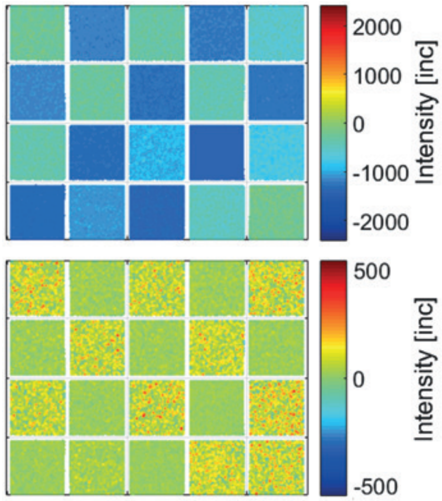

Fig. 3 | Intensity distribution of a target (top) and intensity deviations to the mean intensity value of respective segment (bottom)

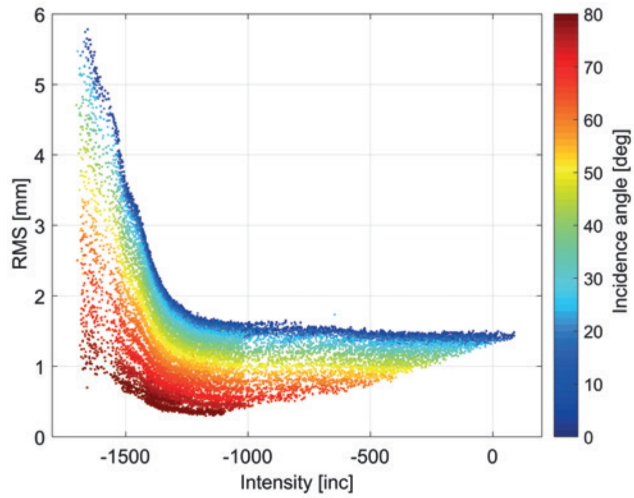

Fig. 4 | Intensity versus root mean square of best plane fit for segments at various incidence angles (coloured) and distances

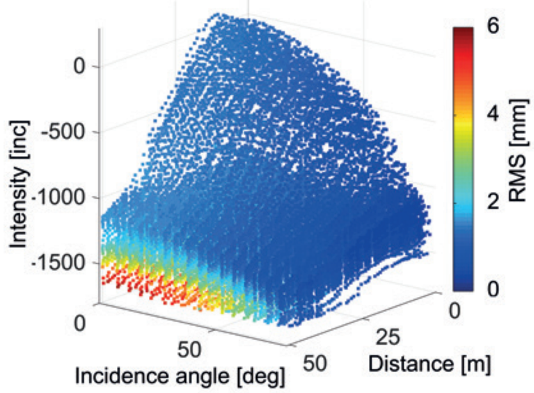

Fig. 5 | Root mean square (coloured) of fitted planes at various incidence angles and distances versus intensity values

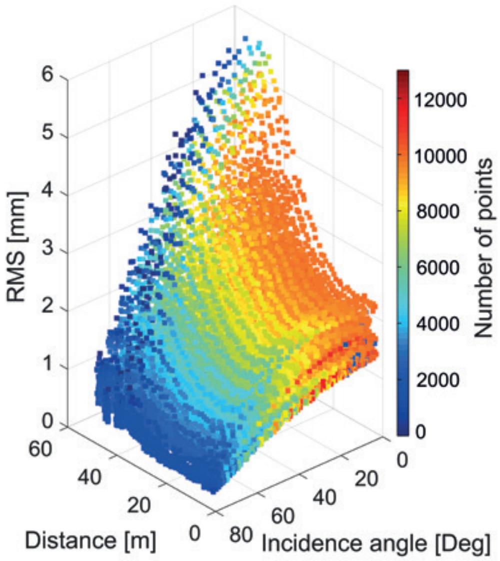

Fig. 6 | Number of points (coloured) for fitted planes at various incidence angles and distances versus root mean square

In *Fig. 6* the RMS is shown at various incidence angles and distances where the colour indicates the number of scan points per segment. As known the number of points decrease notably with increasing incidence angles. For longer distances the returning signals for dark segments were too weak to be recorded reliably from the laser scanner which explains the small number of points (here in blue) above a distance of 50 m. The poorest RMS is given for segments located perpendicularly and furthest away from scanner. As already shown in previous figures, higher incidence angles lead to an improvement of precision. A possible explanation can be found when focusing on the measurement positional uncertainty for single laser scan 3D points as described in the next section.

4 UNCERTAINTY OF THE SAMPLED POINT

When a laser beam hits an object it produces a footprint on its surface. While for perpendicular scans the footprint is circular, the laser beam forms an elongated ellipse for higher incidence angles. This leads to more spreading of the energy distribution and hence a weaker signal return. The SNR increases with the cosine of incidence angle and also inversely proportional to the squared distance /Soudarissanane et al. 2011/. Nevertheless, the RMS of the estimated plane segments in our experiment contradicts this at least for higher incidence angles and requires more investigation into error-propagation. /Bae et al. 2007/ formulated an explicit form of positional uncertainty for 3D laser scan points. For clearness it should be mentioned again that the term positional uncertainty is adopted from /Bae et al. 2007/ and used in our contribution to describe the uncertainty of a single 3D point based on theoretical calculations as introduced in /Bae et al. 2007/. These values will serve in the following for evaluation purposes in our empirical scan experiment concerning the precision of scan points. With known range and angular uncertainties provided e. g. based on the manufacturer specifications the uncertainty of a 3D point in the laser beam coordinate system can be expressed using Variance-Covariance Propagation (VCP). These values can be visualised e. g. in 2D as an error ellipse. In order to obtain the uncertainty in the object's surface normal direction the error ellipse is transformed in respective object's surface coordinate system for instance by a similarity transformation. *Fig. 7* shows an example of an error ellipse at 10 m, 67 m and 100 m distances as well as incidence angles of 0°, 40° and 80°. For simplification a 2D ellipse (1 σ) in the horizontal direction was plotted, that is also expandable for three dimensional space as provided in previous mentioned publication. In this example the theoretical uncertainty is set to 12 seconds for angles (which is also the beam divergence of the applied laser scanner) and 4 mm for distances being the typical observation uncertainty values in common TLS applications provided by the manufacturer /Leica Geosystems 2012/. The horizontal dashed line represents the object's surface and the black line the incoming laser beam hitting the surface. The blue error ellipse indicates the theoretical positional uncertainty for the respective scan point. When focusing on various distances an expansion of the error ellipse in the surface tangent direction is visible (*Fig. 7a, 7d, 7g*) as also the footprint increases linearly in relation to the distance.

However, far more interesting is the rotation of the error ellipses for non-zero incidence angles. Since the impact of the uncertainty of the range measurement is poorer than the resulting angular uncertainty for shorter distances the error ellipse aligns its longest axis parallel to the laser beam, when the incidence angles become non-zero (*Fig. 7c*). Based on theoretical calculations in our example, at a distance of around 67 m the influence of angular uncertainty is equal to the uncertainty of the range measurement and results in a circle (*Fig. 7d, 7e, 7f*). For distances of over 67 m the magnitude of angular uncertainty exceeds the uncertainty of range measurement, hence the shorter ellipse's axis is aligned towards the scanner in laser beam direction (*Fig. 7g, 7h, 7i*).

Considering the alignment of these theoretical error ellipses from /Bae et al. 2007/ the correlation between incidence angle and resulting RMS from the best plane fit for segments in our empirical experiment can be explained as follows: For shorter distances the range measurement is the least precise and most noisy observation. In this case, the highest noise level is found in the laser beam direction, when examining the size of the theoretical error ellipse as an indicator for the noise level. Hence, the noise in surface normal direction reaches here its maximum for perpendicular incidence angles and improves for scans with non-zero incidence angles due to the geometry of the error ellipse. *Fig. 8* outlines two cases, a theoretical error ellipse with an incidence angle at 10° (left) as well as for an incidence angle at 80° (right) where the distance remains equally at 20 m. The green line indicates the supposed noise level in object's surface normal direction and the black line the incoming laser beam.

In order to compare these supposed noise levels with a corresponding target of our scan experiment, the noise of a target scanned at 10° (left) and 80° (right) with a distance of 20 m is shown in *Fig. 9*. When comparing the magnitude of noise of *Fig. 8* and *Fig. 9*, the chosen uncertainties for VCP based on /Bae et al. 2007/ might be slightly too optimistic. Both theoretical error ellipses state a better result than the actually observed noise in the

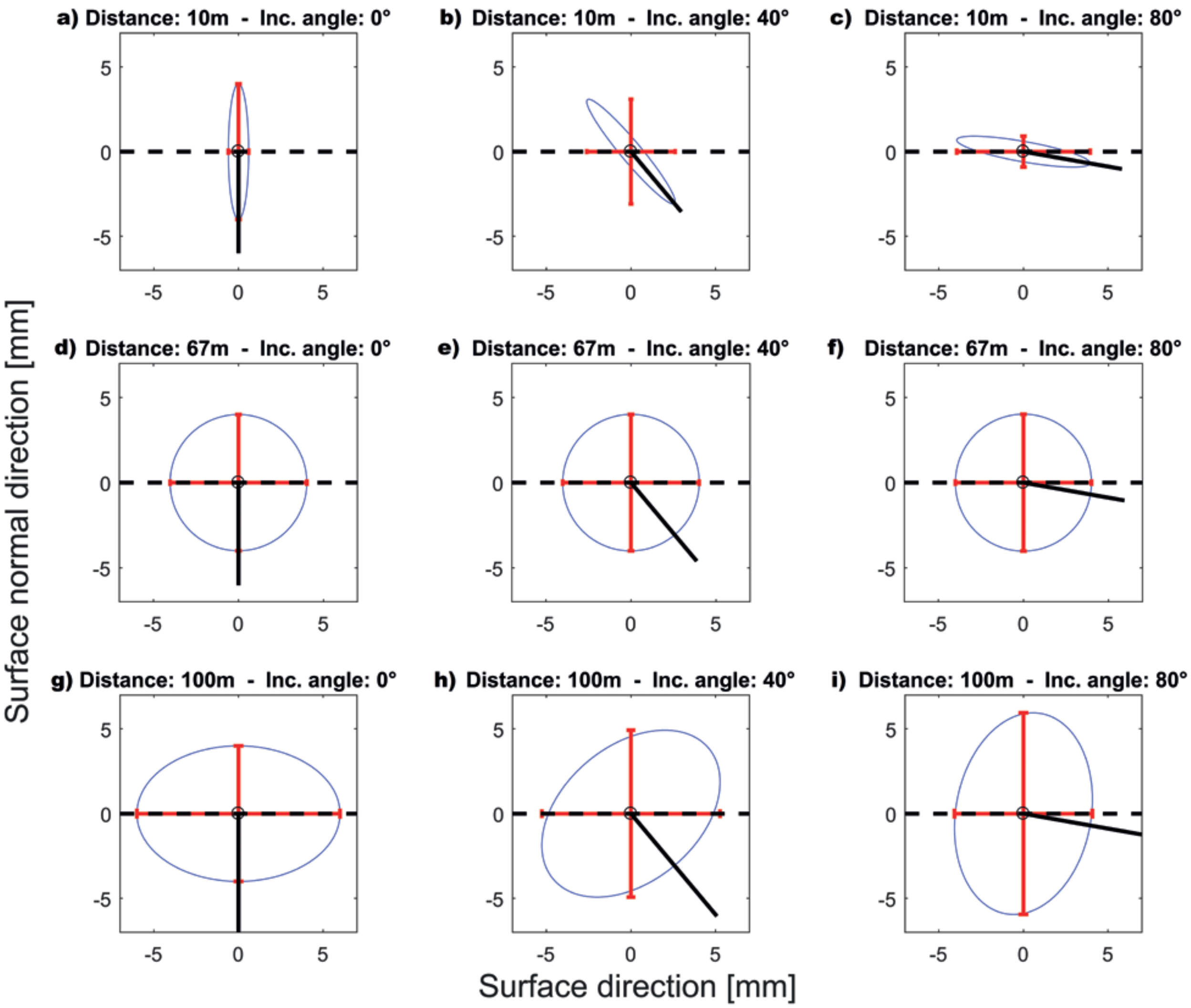

Fig. 7 | Theoretical error ellipses (in blue) of scan points on object's surface (dashed lines) for various distances and incidence angles where the red lines present the uncertainty in surface tangent as well as surface normal direction and the black lines indicate the incoming laser beam

segments, since the theoretical error ellipse does not include further influences e. g. surface properties. Nevertheless, the proportion between both noise levels in *Fig. 8* and *Fig. 9* is the same, specifically the precision deteriorates around factor 2 for scans from 80° to 10°. Additionally, a slight negative trend of noise for the target scanned at 80° is visible. This can be caused by systematic influences due to an increasing incidence angle from the left to the right target edge and hence enlarged footprints.

5 FUNCTIONAL RELATIONSHIP BETWEEN INTENSITY, INCIDENCE ANGLE AND PRECISION

The data of all segments were used to estimate a functional model between intensity I, incidence angle θ and precision defined by the RMS in the form of

$$RMS = f(I) \times g(\theta), \tag{4}$$

with the term $f(I)$ which represents the precision in range direction σ_r and the term $g(\theta)$ the contribution due to the incidence angle θ.

With regard to *Fig. 10* the RMS in surface normal direction can be expressed as

$$RMS = \sigma_r \cos(\theta). \tag{5}$$

This equation is similar to the first equation term of the result from /Bae et al. 2007/ where the variance in the normal direction $\hat{n}$ with range r and incidence angle θ is defined as a function of

$$\sigma_{\hat{n}}^2 = \cos^2(\theta)\, \sigma_r^2 + \sin^2(\theta)\, r^2 \sigma_\theta^2. \tag{6}$$

Note that this equation is based solely on the geometry. It does not explicitly take into account that the surface properties (e. g. spectral reflectance) have an effect on the variance of the range measurement. /Bae et al. 2007/ assume that σ_r is constant or supplied by the manufacturer. It has been demonstrated in Section 3 that the noise level increases as the intensity level decrease which is taken into account by σ_r being approximated through the intensity value. For this paper, the second term is ignored because it is assumed that $\sigma_\theta = 0$ since the precision of 3D points mainly depends on precision of the reflectorless distance measurement as stated in Section 1.

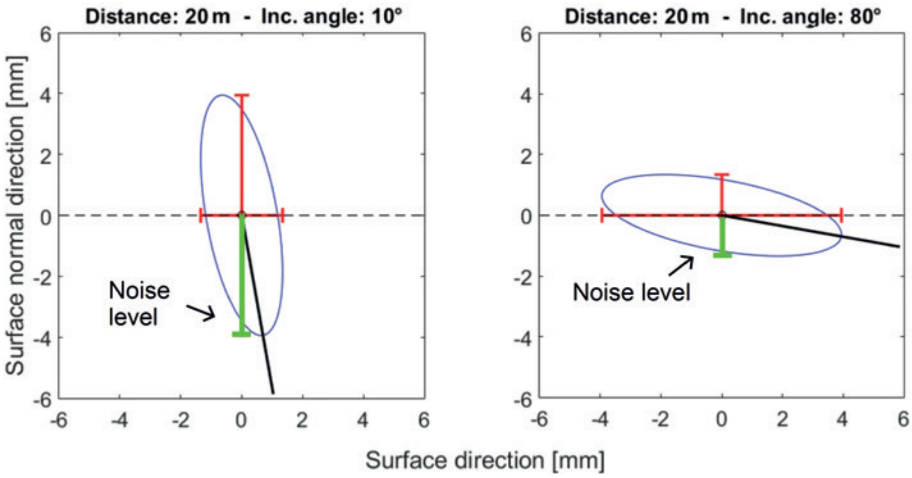

Fig. 8 | Theoretical error ellipses showing the supposed noise level in object's surface with low incidence angle (left) and high incidence angle (right) at 20 m distance

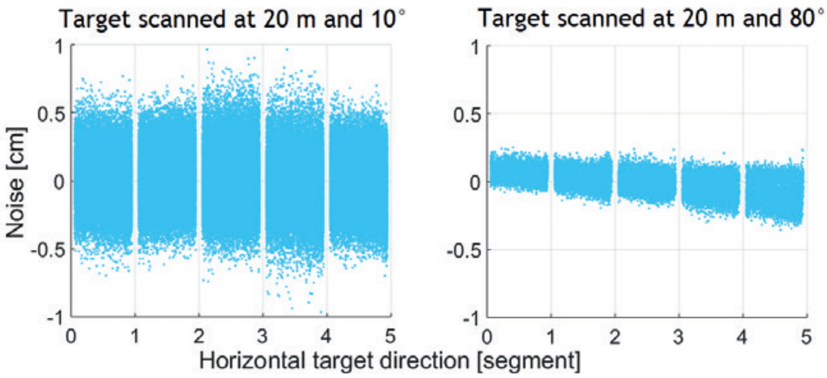

Fig. 9 | Noise level for a target scanned at an incidence angle of 10° (left) and 80° (right) at 20 m distance showing an improvement of precision due to scan geometry

By combining both terms, an approximation for the RMS based on intensity can be given as

$$RMS = (a_0 \cdot e^{(a_1 \cdot I)} + a_2)\cos(\theta). \quad (7)$$

The parameters for this model were solved by least-squares adjustment; similar solving techniques are also applicable. As the least-squares adjustment is non-linear and very sensitive to the model a good initial approximation of the parameters has to be chosen so the adjustment converges. These initial parameters were estimated by the direct search method, i. e. in an iterative process the approximated parameter values were varied by a certain step size. The best fit resulting parameter set was retained and the iterative process repeated using these new parameters and half of previous step size until the step size becomes insignificant. Afterwards, least-squares adjustment was applied to refine the solution using the intensity values I and incidence angles θ as equally weighted observations. Within 20 iterations the solution converged and leads to parameters shown in *Tab. 1*.

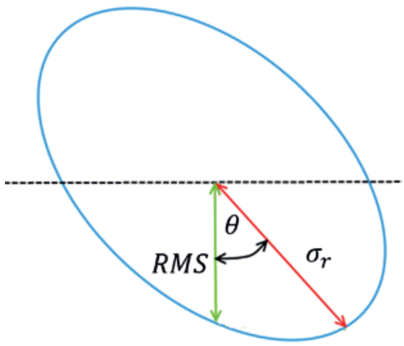

Fig. 10 | Theoretical root mean square of single point calculated in surface normal direction

The derived stochastic model with adjusted parameters is shown in *Fig. 11*. It can be seen that the RMS improves slightly with increasing incidence angles but also deteriorates for lower intensity values.

When comparing the RMS calculated from the stochastic model as given Eq. (*7*) with the observed RMS derived from the best plane fit as described in Section 2 the standard deviation (1 σ) is 0.12 mm and the mean residual between observed and calculated RMS less than 0.01 mm and hence, confirms this model. With known intensity value and incidence angle the stochastic model enables to approximate the precision of each 3D scan point in surface normal direction of an object.

Restricting the stochastic model to an incidence angle of zero, the RMS becomes equal to the precision in range direction as shown in Eq. (*8*). It follows an exponential function with the RMS solely depending on the intensity value:

$$RMS = (1.4 \cdot 10^{-7})\, e^{-0.0064\, I} + 0.0014. \quad (8)$$

Fig. 12 shows the adjusted function (Eq. (*8*)) for RMS (in black) when using perpendicular scans only. Additionally, the empiric results of single scan segments are plotted and the points colourised depending on distance. These results are equal to *Fig. 4* considering the data for incidence angle $\theta = 0$ (blue points in *Fig. 4*).

	Parameter model	Standard deviation of parameter (1 σ)
a_0	0.000 000 40	8.2e–09
a_1	0.005 68	1.3e–05
a_2	0.001 402	2.0e-06

Tab. 1 | Estimated parameters of the stochastic model considering intensity values and incidence angles

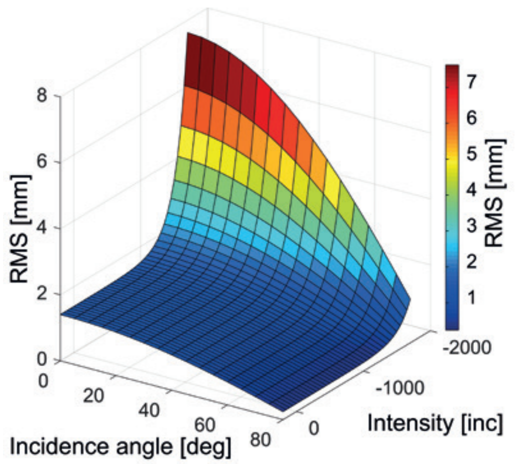

Fig. 11 | Stochastic model as function of intensity and incidence angle

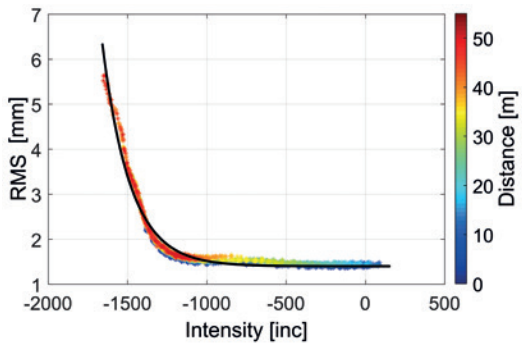

Fig. 12 | Empirical results and adjusted function (in black) of the stochastic model for perpendicular scan segments

6 DISCUSSION

The results of the empirical experiment show a strong correlation between the observed intensity and the resulting precision of scan points. The weaker the signal strength, the higher the RMS of best plane fit. Regarding the fact that the precision of scan points is a combination of numerous influences and difficult to model due to its complexity the intensity presents a meaningful value for stochastic information.

However, in nature often objects with rough surfaces are given yielding to a higher level of obtained noise. Then the derived precision of scan points would be a combination of noise in measurement as well as "roughness noise" in surface. With intensity it is possible to estimate the noise provoked e.g. by surface properties and scan configuration; the intensity cannot indicate the fitting of a single spot on the objects' surface in comparison to surrounding area in terms of modelling objects or surfaces.

Additionally, the noise level in surface normal direction also depends on the incidence angle. Particularly for nearby objects, high incidence angles lead to an improvement of precision in surface normal direction. In this case the noise in the range measurement direction which is the least precise observation for short distances has less influence to the noise in surface normal direction.

The expression of positional uncertainty for 3D scan points introduced by /Bae et al. 2007/ is restricted on theoretical uncertainty of laser scanner's observation and does not take influences provoked by e.g. surface properties or troposphere into account. Regarding this the theoretical model cannot be seen as an adequate base for setting up a stochastic model. Nevertheless, it is a suitable indicator for stochastic information for scan points and explains the unexpected results of our experiment, namely an improvement of precision in surface normal direction for higher incidence angles. /Bae et al. 2007/ also show the propagation for theoretical positional uncertainty for various distances and incidence angles. Therein the same settings are used as applied in *Fig. 7* and hence the statement that for distance measurements below 67 m the impact of angular uncertainty is less than the range uncertainty can be made, thus in practice, high incidence angles improve the precision in surface normal direction. Conversely, the magnitude of angular precision exceeds the precision of distances larger than 67 m thus high incidence angles deteriorate the precision in surface normal direction.

In order to determine a stochastic model based on our experiment the precision of scan points in surface normal direction can be described by an exponential function of intensity and incidence angle. The functional relationship was found by a direct search method followed by a least-squares adjustment based on the data of more than 20 000 laser scan segments. In the case that the operator is not able to obtain the incidence angle or is solely interested in the precision in range direction, the stochastic model was also estimated for perpendicular laser scans where the impact of incidence angles is neglected.

It should be highly emphasised that our findings do not disagree that high incidence angles increases the SNR and lead to less accurate measurements and possible systematic influences. A high incidence angle effects by means a far more noisy range observation in the laser beam direction. Furthermore, the signal strength becomes less due to an elongated footprint. The improvement of precision is solely found in the object's surface normal direction. In the case that a rough or inhomogeneous surface is scanned additional errors are provoked e.g. mixed pixels and spreading of signal leading to less accurate results.

Until now the stochastic model does not consider any correlations between points within a point cloud. It might be necessary to investigate into these possible correlations particularly in successive observations, i.e. following points in a line. In /Kauker et al. 2016/ a short introduction in correlations of laser scanning point clouds was given and a synthetic covariance matrix was proposed but it was also stated that until now spatio-temporal correlations are mostly unknown. However, this aspect is left up to further research.

It is well know that TLS devices are often affected by systematic errors e.g. /Holst et al. 2016/. Our stochastic model is not able to take influences provoked by systematic errors e.g. instrumental imperfections or atmospheric effects into account since the intensity value is solely an indicator for the precision of the distance measurement. Therefore it is highly recommended to perform a calibration procedure on regular basis and to be aware of this laser scanner imperfectness.

7 SUMMARY

The article at hand confirms the suitability of the recently introduced stochastic model for distance measurements in 1D mode by terrestrial laser scanning based on intensity values by /Wujanz et al. 2017/. In our extended experiment in 3D scanning mode a similar relationship between intensity and RMS is found which can be expressed as an exponential function for perpendicular scans. Hence, the intensity can be seen as adequate value in order to derive stochastic information for the precision of range measurements where surface properties and scan configuration are considered. Nevertheless, it was shown, that the scan geometry particularly incidence angles have a notable improving influence in the precision of the surface normal direction and should not be ignored. In many applications such as deformation monitoring or 3D modelling, the emphasis for precision is focused in the surface normal direction and hence by far of more interest than the measurement precision in laser beam direction. In order to tackle this problem an extended stochastic model including intensity values and scan geometry in 3D space is required. The RMS of a scan point in surface normal direction is given as a function of intensity and incidence angle. This stochastic model was estimated based on the data of 20 000 laser scan segments and an explicit function presented. Additionally, the function is given in a simplified form restricted to incidence angles of zeros (perpendicular scan direction). Further research will focus on validating the generality of the model and the applicability for other scanners than the Leica C10 laser scanner.

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