# Measurement of Displacement and Deformations on the Biggest Slovenian Viaduct, with Particular Stress on Accuracy Calculations

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## 1 Introduction

Control measurements can be performed in a variety of ways depending on the structures. In practice, control measurements are performed with the help of geodetic measurements, the basic goal of which is to capture any geometric changes in the measured object. Displacements and deformations are determined. This means defining the position of changes and the object's shape, with respect to the environment and time. In this way, data about the safety of buildings can be obtained to study their behaviour, with the aim of improving the design of similar objects in the future. The main goals of geodetic control measurements are:

- to obtain a certificate for the safe operation and the stability of the measured building,
- to capture geometric changes in the measured object over time,
- to gather data for understanding the causes and creation of changes in geometric attributes,
- to enable predictions for the likely behaviour in the near future and the behaviour under a determined load,
- to control the material characteristics and nature of structures to better model constructional behaviour,
- to gain experience or knowledge in the future, for the planning of similar constructions or their restoration.

The basic tasks of the control measurements that are performed on buildings are defining displacements in horizontal and vertical planes and changes in the geometric shape of that object, such as translation, rotation, twisting, shear, bending and torsion (see Figure 1).

Every road bridge longer than 15 m and every rail bridge longer than 10 m have to be checked (burdened) by legislation in Slovenia [1]. Vertical displacements and different specific deformations have to be measured.

This article concentrates on the measuring of vertical displacements and deformations, the determination of the measurement accuracy and on the analysis of results, of the biggest Slovenian road viaduct Črni Kal.

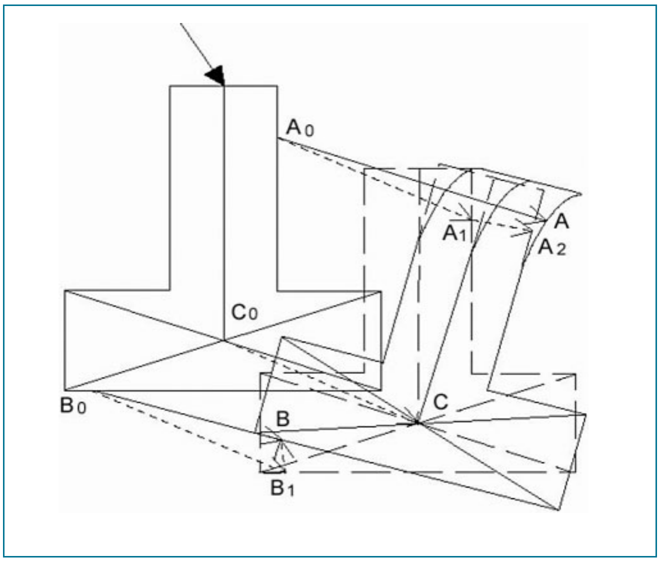


*Fig. 1. Possible changes of the geometric shape of an object*

## 2 Building Description

The Črni Kal viaduct is the most sophisticated bridge construction in Slovenia regarding its functional demands. These include its constructional and technological components, its intricate design and construction regarding the preservation of the environment and the costs of construction and maintenance. It is also the biggest and highest bridge on the Slovene road network (1065 metres in length and 95 metres in height).

The lower part of the viaduct features two girders (7.5 m high) and 11 columns (see Figure 2), five of them are low, double columns, up to 27 m high, six of them are high single columns, that are, in the surfaces part, y-shaped (see Figure 3). The highest column is 87.5 m. The greatest span between columns is 140 m. The total width of the two (separated) roads is 26.5 m.

The columns of the viaduct were built with self-climbing formwork, this technology being used for the first time in Slovenia. The connection of the upper construction was necessary in order to concrete the connecting segments on the left and right sides at the same time. The viaduct

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*Fig. 2. Črni Kal – a view from Osp valley*

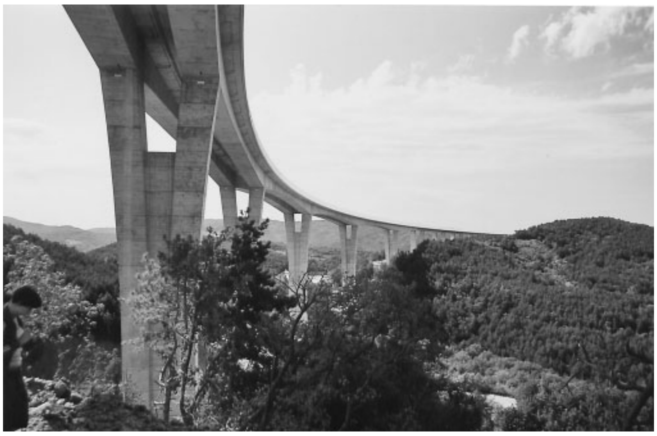

*Fig. 3. "Y" shape of the highest columns*

contains approximately 50,000 cubic metres of concrete, 8,000 tonnes of reinforcing steel and 1,300 tonnes of prestress cable.

After a precise study of the surface characteristics, climatic conditions, configurations in the field and expected strengths of eventual seismic waves, the Engineering Bureau commissioned the design the viaduct. This demanding project necessitated combining theoretical knowledge of civil engineering, mathematics and mechanics and also many years' of experience in the field of building technology.

Plans were made for every part of the construction and for every building phase. Detailed static calculations and analysis of the dynamic responses of this construction were performed. The design was also tested in an Austrian wind tunnel in Vienna (see Figure 4 and 5).

To control the viaduct's behaviour during construction, the concrete's temperature was measured. This required the installation of a considerable number of temperature-measuring devices (Figure 7). Figure 6 shows the increase in concrete temperature in the early stages of the construction followed by a fall in temperature over 14 days. The temperature is highest in the middle part of the foundation plate (130 cm) while the temperature in the deeper section of the foundation plate (280 cm) is almost constant, and falls very slowly.

Ponting d.o.o., a structural engineering company from Maribor, saw that the viaduct needed load testing. They prepared an expert's report of the maximum torques for each field (a span between two columns), which also captured the calculated analysis of constructional behaviour. Separate analyses for each field and for each driven construction (both for left and right) were prepared. The calculated maximum vertical displacements are shown in Table 1.

The quantity of load for a load test was determined by using a model of the spatial framework (with a computer program). The mathematical model of the construction was identical to the model first used in the project.

## 3 Results

Geodetic measurements were performed simultaneously on six station points at the same time (because of the time available). The measurements were made from concrete pillars that were reinforced with steel reinforcement bars and anchored to the ground. These observation pillars were prepared 14 days before measurements so that the material could consolidate well. Also, a net of geodetic points within a local coordinate system was established (two parts because field constraints – from Column 1

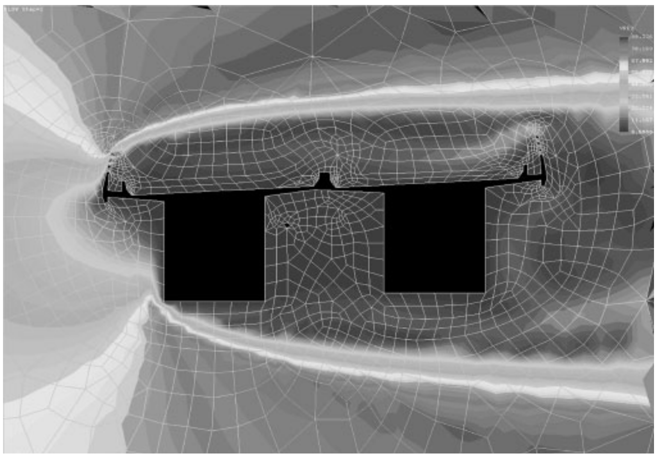

*Fig. 4. Analysis of wind impacts on viaduct*

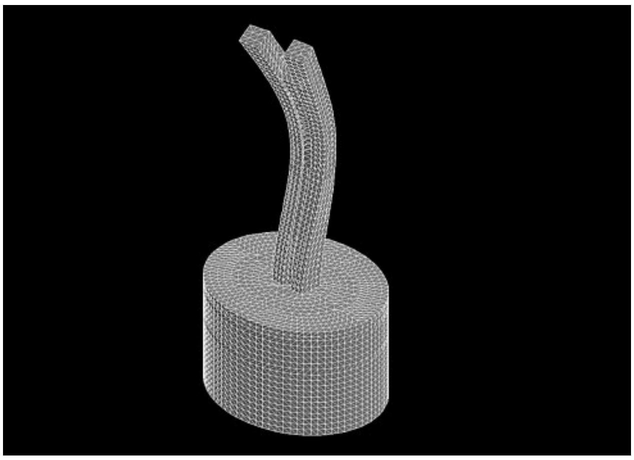

*Fig. 5. Dynamic analysis of support columns-bending*

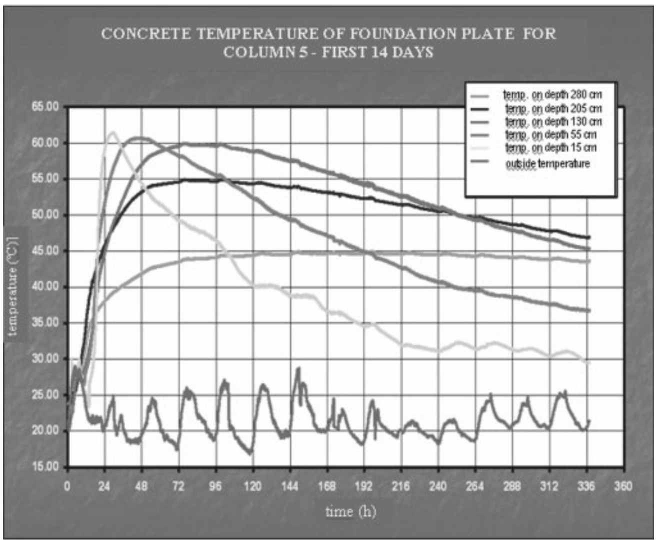


*Fig. 6. Movement in the concrete's temperature*

*Fig. 7. Temperature measuring device attached to steel reinforcement*

*Table 1: Calculated maximum vertical displacements by fields in mm*

| Field | Road direction | | | | | |
|---|---|---|---|---|---|---|
| | towards Koper | | | towards Ljubljana | | |
| | Pylon on KP[1] side | middle-field | Pylon on LJ[2] side | Pylon on KP side | middle-field | Pylon on LJ side |
| 1 | – 0,01251 | – 11,20158 | – 0,15435 | – 0,01074 | – 29,77986 | – 0,15422 |
| 2 | – 0,37819 | – 35,82911 | – 0,24398 | – 0,41834 | – 38,20297 | – 0,86234 |
| 3 | – 0,54105 | – 44,92177 | – 0,52652 | – 0,16813 | – 43,30919 | – 0,24837 |
| 4 | – 0,73056 | – 47,85397 | – 0,75733 | – 0,33624 | – 45,33493 | – 0,19085 |
| 5 | – 0,34358 | – 46,11753 | – 0,13221 | – 0,54641 | – -44,62204 | – 0,45344 |
| 6 | – 0,71887 | – 30,30640 | – 0,44675 | – 0,22559 | – 28,71912 | – 0,20624 |
| 7 | – 0,02588 | – 13,53882 | – -0,47841 | – 0,19849 | – 13,07834 | – 0,45314 |
| 8 | – 2,42995 | – 8,83822 | – 0,69699 | – 0,60894 | – 8,44403 | – 0,68058 |
| 9 | – 0,64877 | – 5,44881 | – 0,65103 | – 0,63011 | – 5,19805 | – 0,63169 |
| 10 | – 0,55073 | – 5,72538 | – 0,78389 | – 0,53315 | – 5,46094 | – 0,75938 |
| 11 | – 0,57257 | – 5,27300 | – 0,75571 | – 0,55230 | – 5,02672 | – 0,72785 |
| 12 | – 0,41356 | – 3,05030 | – 0,04641 | – 0,39739 | – 2,90707 | – 0,04593 |

[1] KP stand for Koper; a costal Slovenian city
[2] LJ stand for Ljubljana; a capital Slovenian city

to Column 5, and from Column 6 to Column 12). These geodetic points were used as a control points before, during and after each measuring phase.

Prior to each load test, all station point positions were checked for stability. Potential shifts of the columns didn't occur. Before every measurement (in the morning/in the afternoon) all instruments were calibrated and data about temperature and air pressure were entered. First, the zero state was recorded and then one individual phase after another. The bridge was unloaded after almost each test. Nikon series 700 instruments and Leica series 2000 instruments were used for the trigonometric heighting.

Simultaneously, 191 characteristic sight points on the viaduct were observed. Leica's reflective tape targets of dimensions 5 × 5 cm (for closer targets) and 10 × 10 cm (for distant targets) were used on each target point. In Figure 8, the number of targets in the third field and their precise positions in that field are shown (KP – the crossbeam on the Koper side, first 1/4 of the field, 1/3 of the field, the middle of the field and LJ – the crossbeam on the Ljubljana side).

A post-processing of all recorded data was carried out before the analysis (filtering). In this way all errors were eliminated, such as double observation of the same point, wrong order of sightings, etc. The observations were arranged according to individual days and individual station points. Every load test phase was compared (load or relief) with the zero state, which was recorded at the start of each measurement. A comparison of the calculated and measured maximum vertical displacements was made sepa-

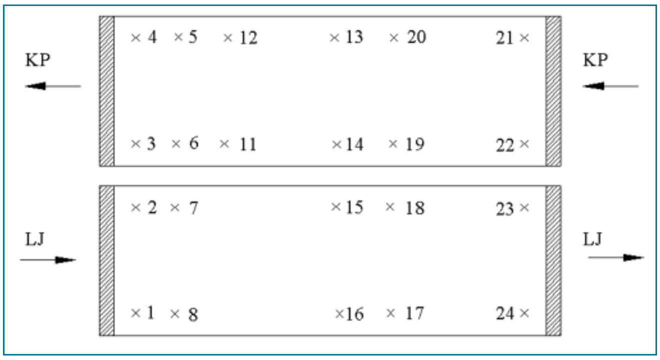


*Fig. 8. Points position in field 3*

rately, for both road ways (left: towards KP, right: towards LJ). Each other's individual profiles were compared; however, in this article only the central and most interesting profiles will be discussed. Figures 9 and 10 shows that the surveyed vertical deformations on all fields, were always smaller then the calculated values. Figure 11 shows the values of the measured maximal displacement during individual phases of the load tests. The common number of phases was 24, the biggest vertical displacement were in phases 15–19, when the longest and highest fields, 3, 4 and 5 were loaded.
The precise processing of the measured data and its analyses were performed after the field measurement.

## 4 Error of altitude difference at trigonometric heights

The accuracy of the measurements was calculated, based on the classical geodetic task of trigonometric heights, where the altitude difference between two points is given by this equation:

$$\Delta H = S_H \cdot \cot Z_A + i_A - l_B + \left(\frac{1-k_a}{2}\right) \cdot \left(\frac{S_H^2}{R}\right) \tag{1}$$

Where:
*$S_H$ – horizontal measured distance between A and B*
*$Z_A$ – zenith distance*
$i_A$ – height of instrument at point A
$l_B$ – height of prism at point B
$k_a$ – coefficient of refraction (for Slovenia $k_a = 0.13$) [3]
R – Earth radium as a sphere ($R = \sqrt{M \cdot N}$; M – radius of curvature along the meridian, N – radius of curvature along the prime vertical (transverse radius of curvature); R = 6370.04 km)

In this case the height of instrument $i_A$ can be omitted from the Eq. (1), because measurements were in a relative coordinate system and all measurement were of a fully local nature. The height of prisms $l_B$ can also be omitted from Eq. (1), because reflective tape targets have negligible thickness.
Equation (1) can be simplified:

$$\Delta H_a = S_H \cdot \cot Z_A + \frac{S_H^2}{2R} - k_a \frac{S_H^2}{2R} \tag{2}$$

The zenith distance can be replaced by the vertical angle ($\alpha$) and slope distance ($D_p$), *so the equation is:*

$$\Delta H_a = D_p \cdot \cos z + \frac{S_H^2}{2R} - k_a \frac{S_H^2}{2R} \tag{3}$$

The slope distance $D_p$, the vertical angle $\alpha$ and the coefficient of refraction $k_a$ are considered as the variables. By using a principles about determining the functions middle errors, function $m(\Delta H) = f(D, \alpha, k_a)$ were obtained

$$m_{\Delta H}^2 = \left(\frac{\partial f}{\partial D}\right)^2 \cdot m_D^2 + \left(\frac{\partial f}{\partial \alpha}\right)^2 \cdot m_\alpha^2 + \left(\frac{\partial f}{\partial k}\right)^2 \cdot m_k^2 \tag{4}$$

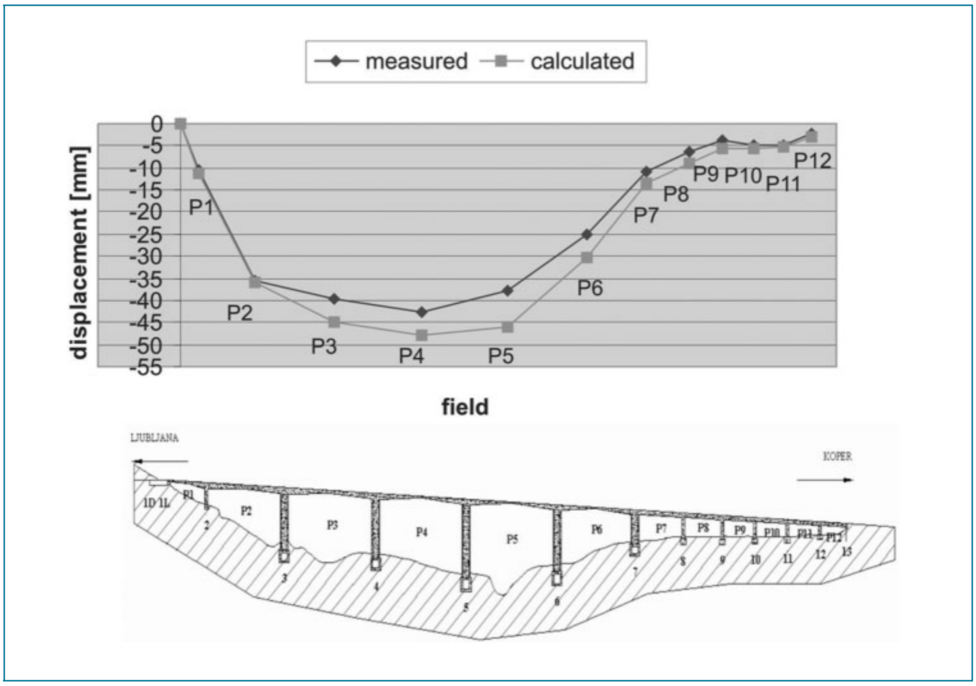


*Fig. 9. Comparison of calculated and measured vertical displacement in the middle of the fields – for left road*

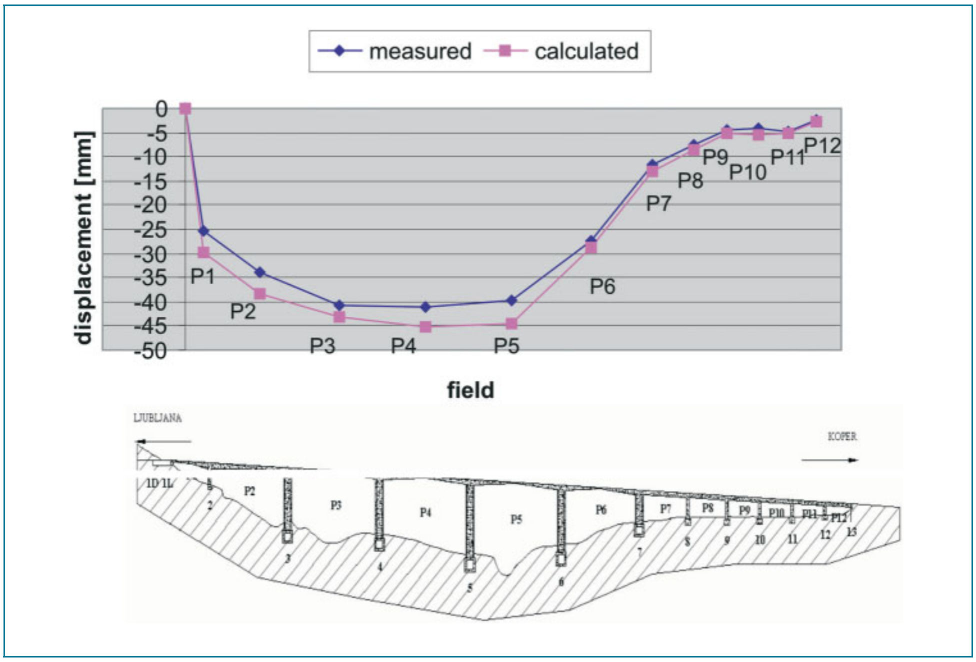


*Fig. 10. Comparison of calculated and measured vertical displacement in the middle of the fields – for right road*

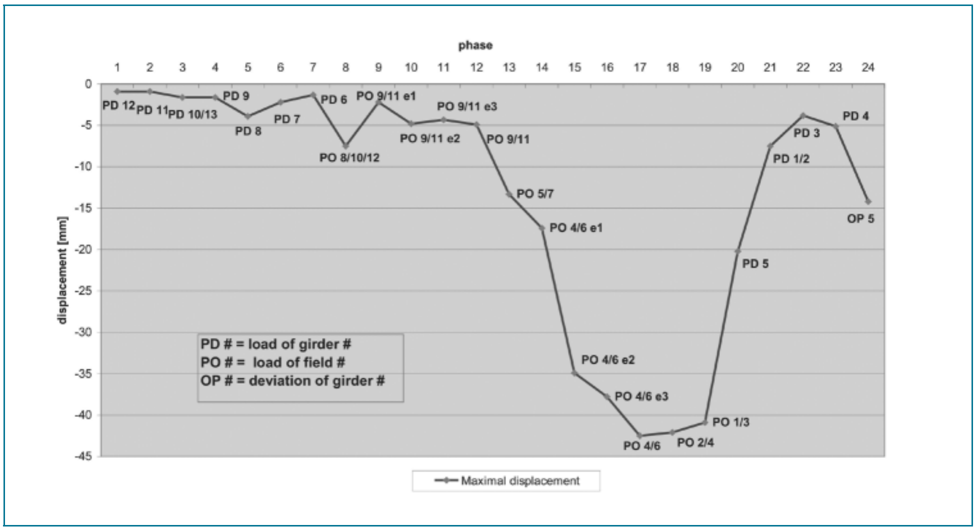


*Fig. 11. Measured maximum vertical displacement by phases*

where:
$m_{\Delta H}$ – standard deviation of height difference error
$m_D$ – standard deviation of distance
$m_\alpha$ – standard deviation of vertical angle
$m_k$ – standard deviation of coefficient refraction error (by pragmatic experiences $m_k = \pm\, 0.05$) [3]
The partial derivatives are:
$\frac{\partial f}{\partial D_p} = \sin\alpha + \frac{D_p}{R} - k_a \frac{D_p}{R}$ – partial derivative for distance, where the last part can be neglected because it is too small
$\frac{\partial f}{\partial \alpha} = D_p \cos\alpha$ – partial derivative for vertical angle
$\frac{\partial f}{\partial k} = -\frac{D_p^2}{2R}$ – partial derivative for coefficient of refraction
The refraction coefficient was considered, despite of the very slope lines of sight. The closer the line of sight to the earth's surface, the bigger is the influence of refraction. Since, the value of refraction coefficient is relatively stable round noon, most measurements were made between $9^{00} - 16^{00}$ h.

The final equation is:

$$m_{\Delta H}^2 = \left(\sin^2\alpha + \frac{D_p^2}{R^2}\right) \cdot m_D^2 + (D_p^2 \cos^2\alpha) \cdot m_\alpha^2 + m_V^2 + m_k^2 \tag{5}$$

In the Eq. (5) the standard deviation of pointing precision $m_V$ was added (calculated bellow). The standard deviations of height differences for different values of slope distance and angles were calculated. A detailed calculation is given bellow for the longest measured slope distance (136.5432 m) and an appropriate vertical angle (55° 13′ 11″); the rest can be seen in Table 2.
In Eq. (5) the standard deviations of distance, vertical angle and refraction were calculated. The accuracy of the distance measurements was taken as ± (3 mm + 2 ppm)

***Table 2: Precision of the height difference for a number of distance-vertical angles pairs***

| No. | $D_p$ [m] | vertical angle $\alpha$ | $m_{\Delta H}$ [mm] |
|---|---|---|---|
| 1 | 5.8474 | 20° 40′ 23″ | 1.06 |
| 2 | 21.5514 | 75° 32′ 53″ | 2.91 |
| 3 | 32.8236 | 55° 18′ 17″ | 2.48 |
| 4 | 50.3400 | 22° 37′ 55″ | 1.34 |
| 5 | 87.0210 | 59° 40′ 20″ | 2.68 |
| 6 | 91.7401 | 28° 54′ 53″ | 1.88 |
| 7 | 114.6949 | 45° 48′ 57″ | 2.47 |

($a$ mm + $b$ ppm), the accuracy of angle measurement as 3″ (from instrument manufacturer specifications).

$$m_D = \sqrt{a^2 + \left(\frac{D_p[m] \cdot b\ ppm}{1{,}000{,}000}\right)^2} = 3.00\ mm$$

$$m_\alpha = \frac{3''}{206{,}264.8062''} = 1.45 \cdot 10^{-5}\ \text{radians}$$

$$m_V = \frac{D_p[mm] \cdot \frac{c}{u}}{63{,}662 mgon} = 0.32\ mm$$

$$m_k = \pm 0.05\ mm$$

The pointing precision $m_v$ was calculated as 0.15 mgon from $\frac{c}{u}$, where $c$ is the resolution of eye (2–8 mgon; mean value is 4.5 mgon) and $u$ is the telescope magnification (in our case 30 ×).

After inserting the values into Eq. (5) and using R = 6370.04 km a final value for the precision of the height difference was obtained:

$$m_{\Delta H}^2 =$$

$$= \sqrt{6.07\ mm^2 + 1.28\ mm^2 + 0.10\ mm^2 + 0.005\ mm^2} =$$

$$= 7.5143\ mm^2$$

$$m_{\Delta H} = \pm 2.7\ mm$$

In this case the precision is mostly influenced by the distance measurement, rather than the pointing, and the measurement of the vertical angles. The precision at other slope distances and angles is shown in Table 2.

## 5 Calculation of inner accuracy

For every target the standard deviation of as distance measurement was also calculated. As an example, only three targets on the longest (and highest) field 4 of the left road are considered, where the biggest vertical displacements were expected.

Ten readings were performed for each target (distance and vertical angle) in the precise measurement mode (PMSR) of the instruments. The arithmetic mean values of the distance readings were calculated. For each target, the standard deviation (see Column 2 in Table 3) and the variance were calculated (see Column 3 in Table 3).

***Table 3. Calculation of standard deviation of slope distances***

| | **point on girder of left construction on LJ side** | | | **point on the middle of the field of the left construction** | | | **point on girder of left construction on KP side** | | |
|---|---|---|---|---|---|---|---|---|---|
| | **reading [m]** | **deviation v [m]** | **squared deviation vv [m]$^2$** | **reading [m]** | **deviation v [m]** | **squared deviation vv [m]$^2$** | **reading [m]** | **deviation v [m]** | **squared deviation vv [m]$^2$** |
| | **1** | **2** | **3** | **1** | **2** | **3** | **1** | **2** | **3** |
| | 0.0020 | 0.0000 | 0 | 0.0412 | – 0,0004 | $1.6 \cdot 10^{-8}$ | 0.0080 | 0.0000 | 0 |
| | 0.0022 | – 0.0002 | $4.0 \cdot 10^{-8}$ | 0.0412 | – 0,0004 | $1.6 \cdot 10^{-8}$ | 0.0082 | – 0,0002 | $4.0 \cdot 10^{-8}$ |
| | 0.0022 | – 0.0002 | $4.0 \cdot 10^{-8}$ | 0.0400 | – 0.0001 | $1.0 \cdot 10^{-8}$ | 0.0083 | – 0,0003 | $9.0 \cdot 10^{-8}$ |
| | 0.0020 | 0.0000 | 0 | 0.0411 | – 0.000 | $9.0 \cdot 10^{-8}$ | 0.0081 | – 0.0001 | $1.0 \cdot 10^{-8}$ |
| | 0.0020 | 0.0000 | 0 | 0.0407 | 0.0001 | $1.0 \cdot 10^{-8}$ | 0.0077 | 0.0003 | $9.0 \cdot 10^{-8}$ |
| | 0.0018 | 0.0002 | $4.0 \cdot 10^{-8}$ | 0.0405 | 0.0003 | $9.0 \cdot 10^{-8}$ | 0.0080 | 0.0000 | 0 |
| | 0.0020 | 0.0000 | 0 | 0.0410 | – 0.0002 | $4.0 \cdot 10^{-8}$ | 0.0084 | – 0.0004 | $1.6 \cdot 10^{-8}$ |
| | 0.0018 | 0.0002 | $4.0 \cdot 10^{-8}$ | 0.0404 | 0.0004 | $1.6 \cdot 10^{-8}$ | 0.0078 | 0.0002 | $4.0 \cdot 10^{-8}$ |
| | 0.0020 | 0.0000 | 0 | 0.0404 | 0.0004 | $1.6 \cdot 10^{-8}$ | 0.0079 | 0.0001 | $1.0 \cdot 10^{-8}$ |
| | 0.0020 | 0.0000 | 0 | 0.0406 | 0.0002 | $4.0 \cdot 10^{-8}$ | 0.0076 | 0.0004 | $1.6 \cdot 10^{-8}$ |
| sum | 0.0200 | 0.0000 | $16.0 \cdot 10^{-8}$ | 0.4080 | 0.0000 | $9.2 \cdot 10^{-7}$ | 0.0800 | 0.0000 | $6.0 \cdot 10^{-7}$ |
| mean | 0.0020 | | | 0.0408 | | | 0.0080 | | |
| variance $\sigma$ | $1.7 \cdot 10^{-8}$ | | | $10.2 \cdot 10^{-8}$ | | | $6.6 \cdot 10^{-8}$ | | |
| **standard deviation $s$** | **± 0.13 mm** | | | **± 0.32 mm** | | | **± 0.26 mm** | | |

***Table 4: Comparison of calculated and measured values***

| **Field no.** | **road direction to Koper** | | | | **road direction to Ljubljana** | | | |
|---|---|---|---|---|---|---|---|---|
| | **calculated value [mm]** | **measured value [mm]** | **meas/calc** | % | **calculated value [mm]** | **measured value [mm]** | **meas/calc** | % |
| **1** | – 11.2 | – 10.5 | 0.94 | 93.7 | – 29.8 | – 19.5 | 0.65 | 65.5 |
| **2** | – 35.8 | – 35.4 | 0.99 | 98.8 | – 38.2 | – 33.8 | 0.88 | 88.5 |
| **3** | – 44.9 | – 39.5 | 0.88 | 87.9 | – 43.3 | – 40.9 | 0.94 | 94.4 |
| **4** | – 47.9 | – 40.5 | 0.85 | 84.6 | – 45.3 | – 42.5 | 0.94 | 93.7 |
| **5** | – 46.1 | – 37.9 | 0.82 | 82.2 | – 44.6 | – 39.7 | 0.89 | 89.0 |
| **6** | – 30.3 | – 24.9 | 0.82 | 82.2 | – 28.7 | – 27.4 | 0.95 | 95.4 |
| **7** | – 13.5 | – 11 | 0.81 | 81.2 | – 13.1 | – 11.5 | 0.88 | 87.9 |
| **8** | – 8.8 | – 6.5 | 0.74 | 73.5 | – 8.4 | – 7.5 | 0.89 | 88.8 |
| **9** | – 5.4 | – 3.9 | 0.72 | 71.6 | – 5.2 | – 4.5 | 0.87 | 86.6 |
| **10** | – 5.7 | – 4.7 | 0.82 | 82.1 | – 5.5 | – 4 | 0.73 | 73.2 |
| **11** | – 5.3 | – 5 | 0.95 | 94.8 | – 5.0 | – 4.9 | 0.97 | 97.5 |
| **12** | – 3.1 | – 2.1 | 0.69 | 68.8 | – 2.9 | – 2.4 | 0.83 | 82.6 |
| | | | **average** | **83.5** | | | **average** | **86.9** |

The variance was calculated: [2]

$$\sigma^2 = \frac{\sum v^2}{(n-1)} \tag{6}$$

and the standard deviation by equation:

$$s = \sqrt{\frac{\sum v^2}{(n-1)}} \tag{7}$$

where:
$s$ – standard deviation,
$v^2$ – squared deviation from arithmetic mean
$n$ – number of observations.

Table 3 shows the calculation of the standard deviation for field 4 (the crossbeam on Ljubljana side (LJ), the centre of the field, the crossbeam on Koper side (KP)).

A measurement precision of $\pm$ 0.2 mm was achieved (average value of the standard deviation of the distance measurement to targets near pylons) and $\pm$ 0.3 mm (average value of the standard deviation of distance measurements to middle of field).

In Table 4 the calculated and measured values of the deformation of the middle points of both roads are shown. The measured vertical displacements for the road direction to Koper, is between 68.8 % and 98.8 % (average 83.5 %). For the road direction to Ljubljana, the measured values are between 73.2 % and 97.5 % (average 82.9 %) of the calculated value. In practice there is an unwritten rule that the ratio should be around 75 %. If we compare only those ratios for the smallest vertical displacement (because they are the most questionable) for the two parallel road ways, that is 68.8 % and 82.6 %, a ratio that is almost exactly as expected ((68.8 % + 82.6 %)/2 = 75.7 %). We conclude that a proper measurement method was selected.

## 6 Conclusion

Today, the method of trigonometric heighting is the most commonly used for determining the vertical displacement with total stations. The advantage of this method is that on longer sighting distances than in levelling can be used. In addition to vertical displacements, horizontal displacement can also be obtained. The measurements are quicker and, most important, partial results can already be obtained in the field. So the main remaining problem relate to the operator, such as the errors that occur because of inaccurate pointings.

The method used for the precision testing of the Črni Kal viaduct was suitable, because the measurement precision in comparison to the magnitude of the measured vertical displacement was sufficient. Six instruments (total stations) of the similar accuracy were used. Every measurement, was repeated 10 times which turned out to be appropriate. The reflective tape targets were big enough.

The measured values of the vertical displacements of the fields increased with the field length, as expected.

For large load tests, as in the case of the Črni Kal viaduct, GPS receivers could be also used. Levelling was not suitable at all. When executing combined measurements (with total stations and GPS) one would get similar results; but because of the results would be expected to be similar accuracy, combined measurements would be pointless.

## References

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## Abstract

**A lot has been written and said about the measuring of displacements and deformations of structures. And yet, in practice something new and useful is found at every bigger and more complicated construction. This article introduces displacement measurements on the largest viaduct in Slovenia, and an analysis of the results, with particular stress on the accuracy of the calculations. Today there are a lot of sophisticated methods to measure and analyse a bridge during load tests as GPS, photogrammetric measurements, laser scanning, levelling with digital or laser levels, etc.. Nevertheless, the use of classical techniques such as trigonometric heighting is still good enough for the most demanding field observations. In this article the use of trigonometric heighting during a load test on the biggest Slovenian viaduct „Črni Kal“ is discussed.**