

# Design of a Gravity Base Network

Es wird ein Basis-Schwerenetz für den Iran vorgestellt. Die 19 Schwerestationen liegen an nationalen Flugplätzen. An neun Stationen sollen absolute Schweremessungen durchgeführt werden und sechs Stationen bilden eine Eichstrecke vom Nordwesten zum Südosten des Landes.

## 1 Introduction

Following the request of the National Cartographic Center (NCC) of Iran and in a joint project between the NCC and the K.N. Toosi University of Technology, a gravity base network was designed and proposed for Iran. This paper reviews the design process of the network. The network when it is measured will consist of stations with known absolute gravity to serve as a base for future densification networks needed in the country. It is supposed to meet the international gravity standards required also by the international gravity network called the International Gravity Standardization Network of 1971 (IGSN71), MORELLI et al. (1971).

### 1.1 A gravity base network

A gravity base network is supposed to be a set of benchmarks uniformly distributed across the country and the absolute gravity values at the benchmarks are known to the best accessible accuracy. The gravity at the benchmark stations are either measured directly with absolute devices or transferred by gravity difference measurements by gravimeters from known stations. To decrease the accumulation of random measuring errors arising from these transfers, the number of base stations distributed across the country should be as small as possible. This is feasible if the stations are selected in the national airports long distances apart but faster accessible and measurable by a gravimeter carried in an airplane between the stations. The gravimeter used for the gravity difference measurement should be accurate to a few micro Gal, e.g., CG-3/3M, Scintrex L.T.D. (1995), to result in the gravity base stations of required accuracies fulfilling various engineering applications of gravity. To realize the importance of such a network, various applications of a gravity base network are firstly reviewed.

### 1.2 Applications of a gravity base network

A gravity base network is the required reference frame for establishing 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> order gravity networks. As stated in Torge (1989) a gravity network is used for:

- Mapping of the structure of upper crust in geology maps. The required accuracy for the measured gravity values is about 0.2 to 0.4 mGal.
- Oil and mineral explorations. The required accuracy for the measured gravity values is about 5  $\mu$ Gal.
- Geotechnical studies in mining areas for exploring the underground cavities as well as archeological studies. The required accuracy is about 5  $\mu$ Gal and better.
- Subsurface water resource explorations and mapping crustal layers which absorb it. An accuracy of the same level of previous applications is required here too.
- Studying the tectonics of the Earth's crust. Repeated precise gravity measurements at the gravity network stations can assist us in identifying systematic height changes. The accuracy of the order of 5  $\mu$ Gal and more is required.
- Studying volcanoes and their evolution. Repeated precise gravity measurements at the gravity network stations can provide valuable information on the gradual upward movement of lava.
- Producing precise mean gravity anomaly for precise geoid determination. Replacing precise spirit leveling by the GPS leveling using precise geoid model is one of the forth coming application of the precise geoid.

### 1.3 History of the gravimetric surveys in Iran

According to AFSHAR and ZOMORRODIAN (1970), a preliminary First Order Gravity Network (FOGN) in Iran was established using bench marks and points marked on buildings. The instrument used for the measurements was Askania Graf Instrument providing accuracies of  $\pm 0.01$  mGal or better. For the fact that the bench marks were demolished or transferred and also for the rather low accuracy of the net, a new FOGN was established in 1970 in the country. The new network consisted of 23 stations 14 of which were selected in airports for the best accessibility by the air transportation. The other 9 points were located in the railway stations. These 9 stations were selected to accomplish a uniform network. The Mehrabad airport station in Tehran with known absolute gravity was the only reference station defining the datum of network. This station is a component of IGSN71 network measured partly by the US Air Force using La coste & Romberg gravimeter. The over all accuracy of the network was estimated to be  $\pm 0.004$  mGal/km. Fig. 1 illustrates the distribution of the new FOGN network in Iran. In this figure, green stations showing the airports and blue ones are the rest.

In 1970, the Institute of Geophysics of Tehran University (IGTU) established a national calibration line for Iran. The line extended from Shiraz in the south to Chalus in the north of the country. The line covered latitudes

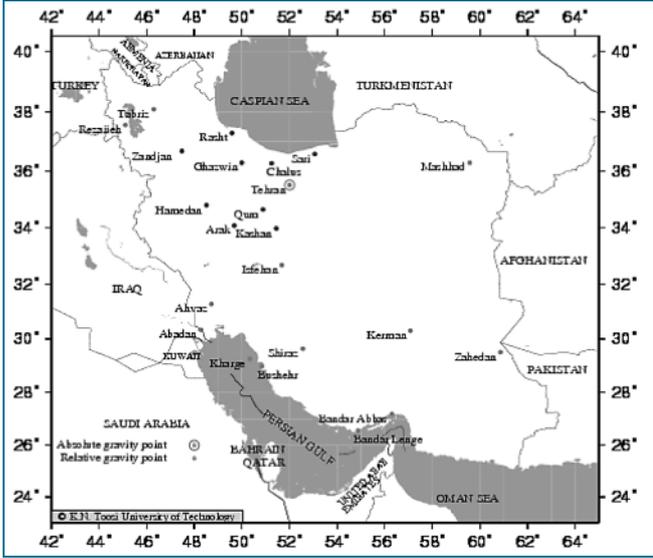


Fig. 1: The first order gravity network of Iran, Afshar and Zomorrodian (1970)

range from  $29.55^\circ$  to  $36.40^\circ$ , ZOMORRODIAN (1972). It was later extended in the south and north directions in a joint project between the National Cartographic Center (NCC) and IGTU, ZOMORRODIAN (1985). In 1987 IGTU established a new gravity datum in IGTU building in Tehran, ZOMORRODIAN (1987).

In 1980, NCC also take hold of gravity measurements in the country. The primary goal was to investigate the systematic effect of gravity field in the leveling network, VANICEK and KRAKIWSKY (1986), in order to correct the orthometric heights in the country. In 1996 NCC bought two sets of CG-3M micro gravimeters. In June 1997 a technical committee was organized in the department of Land Surveying and Geodesy of NCC to promote academic studies on gravity. This committee decided to firstly establish a new gravity base network as a foundation supporting the modern demands:

- i. High accuracy inquiry by the modern gravity projects
- ii. Maintaining high reliability required by integral gravimetric activities
- iii. Inquiry for high resolution and accurate gravity anomaly data needed for a precise geoid.
- iv. Evolution of micro gravimetric networks.

## 2 Computation of a gravity base network

In a gravity network, the absolute gravity values at the stations are sought. Aiming at obtaining absolute gravity with a pre-determined accuracy, a measurement program has to be pre-analyzed. For the gravity difference measurements to result in the absolute gravity values at stations, at least one station is required to be known in its absolute gravity. It is called the reference station. Existence of more than one reference station with known accuracy would help the better adjustment of the network. Assuming gravity differences  $\delta\mathbf{g}$  (vector of observations) measured between the reference stations with known gravity values  $\mathbf{g}^{(0)}$  (vector of initial values) and

the stations with unknown gravity values  $\mathbf{g}$  (vector of unknowns) in the network, two mathematical models one is called the observation equations as the main mathematical model, Eq. (1), describing the relations between the gravity difference measurements and the unknowns, and other as the constraint model, Eq. (2), formulating the known absolute gravity values:

$$\delta\mathbf{g} = \mathbf{A}\mathbf{g}; \mathbf{C}_{\delta\mathbf{g}} \quad (1)$$

$$\mathbf{g}^{(0)} = \mathbf{H}\mathbf{g}; \mathbf{C}_{\mathbf{g}^{(0)}} \quad (2)$$

are performed where  $\mathbf{A}$  and  $\mathbf{H}$  are the corresponding design matrices. Two typical equations making up the main and the constraint models read

$$\delta g_{ij} = g_j - g_i + \Delta t_{ij}d, \quad (3)$$

$$g_k - g_k^{(0)} = 0 \quad (4)$$

respectively, where  $g_i$  and  $g_j$  are the sought absolute gravity values at stations number  $i$  and  $j$ ,  $d$  is the unknown drift of a particular gravimeter,  $\Delta t_{ij}$  is the travel time spent for the measurement,  $g_k$  is the assumed unknown absolute gravity of the  $k$ -th reference station while  $g_k^{(0)}$  is the initially known absolute gravity at the same station. The statistical models  $\mathbf{C}_{\delta\mathbf{g}}$  and  $\mathbf{C}_{\mathbf{g}^{(0)}}$  introduced in Eqs. (1) and (2) are the variance and covariance matrices describing accuracies of the gravity difference observations and the known initial absolute gravity values. The matrix  $\mathbf{C}_{\delta\mathbf{g}}$  is fully populated embedding the variances as well as the covariances between gravity difference measurements obtained from gravimeter readings. The matrix is obtained using the covariance propagation law from the variances of gravimeter readings stored in the diagonal matrix  $\mathbf{C}_R$ :

$$\mathbf{C}_{\delta\mathbf{g}} = \mathbf{B}\mathbf{C}_R\mathbf{B}^T \quad (5)$$

where  $\mathbf{B}$  is the matrix defining the relation between gravimeter readings and the gravity difference measurements. Using weight matrices as the inverse of covariance matrices:

$$\mathbf{P}_{\delta\mathbf{g}} = \mathbf{C}_{\delta\mathbf{g}}^{-1}, \quad \mathbf{P}_{\mathbf{g}^{(0)}} = \mathbf{C}_{\mathbf{g}^{(0)}}^{-1} \quad (6)$$

the least-squares solution to the system of Eqs. (1) and (2) is

$$\hat{\mathbf{g}} = \mathbf{N}^{-1}\mathbf{u}, \quad \mathbf{C}_{\hat{\mathbf{g}}} = \mathbf{N}^{-1} \quad (7)$$

$$\hat{\mathbf{r}}_{\delta\mathbf{g}} = -\delta\mathbf{g} + \mathbf{A}\mathbf{C}_{\hat{\mathbf{g}}}\mathbf{A}^T\mathbf{C}_{\delta\mathbf{g}}^{-1}\delta\mathbf{g} + \mathbf{A}\mathbf{C}_{\hat{\mathbf{g}}}\mathbf{H}^T\mathbf{C}_{\mathbf{g}^{(0)}}^{-1}\mathbf{g}^{(0)} \quad (8)$$

$$\hat{\mathbf{r}}_{\mathbf{g}^{(0)}} = -\mathbf{g}^{(0)} + \mathbf{H}\mathbf{C}_{\hat{\mathbf{g}}}\mathbf{H}^T\mathbf{C}_{\mathbf{g}^{(0)}}^{-1}\mathbf{g}^{(0)} + \mathbf{H}\mathbf{C}_{\hat{\mathbf{g}}}\mathbf{A}^T\mathbf{C}_{\delta\mathbf{g}}^{-1}\delta\mathbf{g} \quad (9)$$

$$\hat{\sigma}_0^2 = \frac{1}{df} \left( \hat{\mathbf{r}}_{\delta\mathbf{g}}^T \mathbf{P}_{\delta\mathbf{g}} \hat{\mathbf{r}}_{\delta\mathbf{g}} + \hat{\mathbf{r}}_{\mathbf{g}^{(0)}}^T \mathbf{P}_{\mathbf{g}^{(0)}} \hat{\mathbf{r}}_{\mathbf{g}^{(0)}} \right) \quad (10)$$

in which

$$\mathbf{N} = \mathbf{A}^T \mathbf{P}_{\delta\mathbf{g}} \mathbf{A} + \mathbf{H}^T \mathbf{P}_{\mathbf{g}^{(0)}} \mathbf{H} \quad (11)$$

$$\mathbf{u} = \mathbf{A}^T \mathbf{P}_{\delta\mathbf{g}} \delta\mathbf{g} + \mathbf{H}^T \mathbf{P}_{\mathbf{g}^{(0)}} \mathbf{g}^{(0)} \quad (12)$$

The vector  $\hat{\mathbf{g}}$  is the least-squares estimate of unknown absolute gravity values including the new values for the reference stations, while  $\mathbf{C}_{\hat{\mathbf{g}}}$  is the corresponding variance-

covariance matrix.  $\mathbf{r}_{\delta g}$  and  $\mathbf{r}_{g^{(0)}}$  are the vectors of the estimated residuals to correct the observations and the initial absolute gravity values.  $\hat{\sigma}_0^2$  is the a-posteriori estimate of the variance factor or the variance of unit weight with df being the degree of freedom in the mathematical model. A statistical test of quadratic forms of the residuals (8) and (9) is carried out to test for the compatibility of the a-posteriori value with its a-priori value. The test is passed if the scale of the covariance matrix  $\mathbf{C}_{\delta g}$  is correct. Rejection of the test may be caused by the

- a. incorrect scale of the covariance matrix,
- b. systematic and/or outlying observations,
- c. inconsistency of the mathematical model and the observations.

It should be reminded that in the method of least-squares, systematic and gross errors may distort the estimate of unknowns. Therefore, controlling the quality of observations both before and after adjustment for the detection of any systematic error and outlier should be necessarily carried out. Nevertheless, depending on the reliability and robustness of a network, outliers that are smaller than a certain limit can cause distortions in the network, i.e., the deformation or distortion of the network is unavoidable. The analysis of these distortions and their quantification is of special interest in the quality control and design of a gravity network, a subject to be followed in this paper.

### 3 Relative gravimeter CG-3M

The relative gravimeter CG-3M is a production of the Canadian company Scintrex, Scintrex L.T.D. (1995). Using an internal microprocessor, the instrument operates automatically. The instrument is capable of measuring gravity variation in the range of 7000 mGal without changing the zero of its counter, from pole to the equator, Fig. 2. Starting measurement, the instrument automatically records the gravity every second. Outlying records are eliminated and replaced by new ones through statistical assessment of the sample of measurements.

Measurements and corrections are stored in the memory of instrument and can be transferred to a computer or printer if inquired. The resolution of the instrument is 1  $\mu$ Gal and its sample standard deviation is normally 5  $\mu$ Gal. The measurements and other relevant information are displayed in the instrument during the measurement. Following table compares the instrument to the CG-3 model and to other types of L&R (LaCoste Rhombberg) gravimeters:

Tab. 1: Accuracy comparison of different gravimeters

Model	Resolution (mGal)	Range (mGal)	Accuracy (mGal)
L&R-G	0.01	7000	0.015
L&R-D	0.001	200	0.005
CG-3	0.005	7000	0.010
CG-3M	0.001	7000	0.005

The accuracy in the table refers to the mean standard deviation of one observation by the instrument inferred from the sample of observation. For further details, the interested reader is referred to the manual of the instrument (Scintrex L.T.D., 1995, SIEGEL, 1995).

### 4 Specifications of a gravity base network

Based on the specifications mentioned above for the CG-3M gravimeter, and to fulfill the requirements of section 1.2 for obtaining an accuracy of better than 5  $\mu$ Gal in the absolute gravity across the network, the geometry of the network, the number of known reference stations, and the number of gravity difference measurement ties should be designed. The number of measurements should be as large as to make the network reliable to detect gross errors in the measurements. Secondly, a robustness analysis is required to design a strong enough network to resist undetectable gross errors, so that the inevitable distortions on the estimated absolute gravity values remain below the desired accuracy. On the other hand, the physical durability of the station benchmarks should be enough to maintain the accuracy of the determined absolute gravity values. To maintain the accuracy of 5  $\mu$ Gal, vertical stability of 2 centimeters in the vertical direction for the benchmarks is required in the life-time of the network. Considering the small horizontal gradient of gravity, the horizontal stability of the stations is easily maintained. Nevertheless, with all pre-cautions spent in construction of the benchmarks, the physical stability of stations is subject to the tectonic movement in the region.

A network of 19 stations to be built in the national airports of Iran is assumed. Airports are usually built in geologically stable places and they are also suitable places for the gravity stations in long distances apart since they are quickly accessible by airplanes. Figure 3 shows a network of totally 19 stations assumed in the airports of Yazd, Tehran, Tabriz, Rasht, Kermanshah, Isfahan, Ahvaz, Abadan, Shiraz, Lar, BandarAbbas, Chabahar, Iranshahr, Kerman, Zabol, Birjand, Mashhad, Kolaleh, and Tabas in Iran. The average spacing between the stations is 460 km, while the minimum distance is 100 km and the maximum distance



Fig. 2: Scintrex gravimeter at a gravity base station in Tehran

is 800 km. Among the 19 stations, 9 stations Yazd, Tehran, Tabriz, Ahvaz, Abadan, Chabahar, Iranshahr, Kerman, and Mashhad are considered as known reference stations, so that the absolute gravity is supposed to be measured at these stations. The station Abadan is located in Arabian plate, Berberian and Berberian (1981). The gravity at this station may have a different behavior in comparison to the other stations of the network. For this reason and despite of the small distance between this and the station Ahvaz, absolute gravity is required to be measured at this station too. The study of the gravity change between these stations can provide valuable information in the tectonics of this area. Today, absolute gravity can be measured to an accuracy of  $3 \mu\text{Gal}$  by FG5, SASAGAWA et al. (1995).

## 5 Design of measurement of the gravity base network

Taking the 9 stations mentioned above as the reference stations, absolute gravity at which are supposed to be measured to the accuracy of  $3 \mu\text{Gal}$  using modern absolute devices, NIEBAUER 1995, SEIGEL (1995), and assuming that the gravity difference measurements are to be done with CG-3M gravimeter whose nominal accuracy is  $5 \mu\text{Gal}$ , different designs of gravity difference measurements between the known and unknown stations have been analyzed. The mathematical tool for the analysis is the relation between the measurement accuracies, the accuracy of the known stations, and the accuracy of unknowns, deduced from Eqs. (5), (7), and (11) as

$$\mathbf{C}_{\hat{\mathbf{g}}} = \left( \mathbf{A}^T (\mathbf{B} \mathbf{C}_R \mathbf{B}^T)^{-1} \mathbf{A} + \mathbf{H}^T \mathbf{C}_{\mathbf{g}(0)}^{-1} \mathbf{H} \right)^{-1}. \quad (13)$$

The matrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{H}$  are defining the geometry of the proposed gravity difference measurements and the known stations. The matrices  $\mathbf{C}_{\mathbf{g}(0)}$  and  $\mathbf{C}_R$  are built up in accordance with the proposed scheme of the measurement and the nominal accuracies of the absolute and relative instruments. To assess the contribution of an observation in the adjustment of the measurements, its redundancy number (VANICEK et al. 1990) among other observations

$$d_i = 1 - \frac{\sigma_{\hat{\mathbf{g}}_i}^2}{\sigma_{\mathbf{g}_i}^2}, \quad (14)$$

has to be investigated. The number can also be visualized as the corresponding diagonal element of the matrix:

$$\mathbf{I} - \mathbf{A} \mathbf{C}_{\hat{\mathbf{g}}} \mathbf{A}^T (\mathbf{B} \mathbf{C}_R \mathbf{B}^T)^{-1}. \quad (15)$$

For the weighted parameters or the initial absolute gravity values too, the redundancy numbers are seen as the diagonal elements of the matrix:

$$\mathbf{I} - \mathbf{H} \mathbf{C}_{\hat{\mathbf{g}}} \mathbf{H}^T \mathbf{C}_{\mathbf{g}(0)}^{-1}. \quad (16)$$

Five different sets of gravity difference observations were proposed for the network measurement. At each set, a nuisance unknown parameter of the gravimeter called drift is also considered. To solve for the daily drift rate, multiple forward and backward running of gravimeter measurements are required. Each set of observations was analyzed

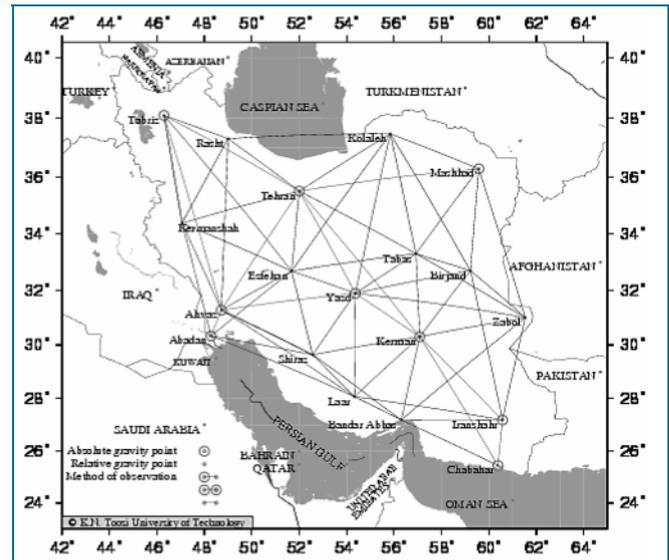


Fig. 3: The proposed gravity base network of Iran with gravity difference measurements ties

for the internal and external reliability assigns to the network in accordance with the reliability theory of BAARDA (1976). In his reliability theory, BAARDA (1976) uses the concept of alternative hypothesis test, WONNACOTT and WONNACOTT (1977), to determine how a network is reliable in detecting a blunder(s) among its observations. BAARDA measures the reliability in terms of the power of alternative test  $1 - \beta$ ,  $\beta$  being the probability of type II error, conducted on each observation in the network. That is, a network is  $(1 - \beta)\%$  reliable to detect a blunder equal to or larger than a certain size  $\Delta_i$  in an observation. This means that, the blunders or gross errors smaller than  $\Delta_i$  may remain undetectable in the network. This threshold gross error is shown to be a function of measured accuracy, redundancy number of the observation, and is proportional to the non-centrality parameter  $\delta$  of the alternative hypothesis test as well. The parameter itself is a function of the significance level  $\alpha$ , i.e., complementary to the confidence level of the null hypothesis test, and the  $\beta$ . Opting for  $\alpha = 0.05$ ,  $\beta = 0.10$ , and for the statistic being the normal standard, the non-centrality parameter  $\delta$  would be equal to 3.24. The threshold gross error is then given by

$$\Delta_i = \frac{3.24\sigma_i}{\sqrt{d_i}} \quad (17)$$

where  $\sigma_i$  is the standard deviation of the observation number  $i$ , and  $d_i$  is the redundancy number. According to the equation, an observation with smaller standard deviation and larger redundancy number leaves smaller gross error undetected in the network. To keep all undetected gross errors smaller so that the caused gravity distortions in the network drop under a desired value, the stations are required to be evenly distributed across the network in a way to keep the travel times between stations almost the same, and the observations are repeated as frequently as possible, BECKER (1995). The largest value of  $\Delta_i$  computed across is known as the internal reliability of the network at the 90% confidence level. Observational errors smaller than this value are assumed to remain undetected.

Hence, they may cause virtual as such unwanted distortions in the network. The amount of distortions at the network stations are computed by the following equation

$$\Delta \hat{\mathbf{g}} = \mathbf{N}^{-1} \Delta \mathbf{u} = \mathbf{N}^{-1} (\mathbf{A}^T \mathbf{P}_{\delta g} \Delta_{\delta g} + \mathbf{H}^T \mathbf{P}_{g^{(0)}} \Delta_{g^{(0)}}). \quad (18)$$

This is known as the external reliability of the network. This equation gives the change or distortion in the gravity value of the stations. Using Eq. (18), distortions in the network initiated from each of the five proposed set of observations are computed. The optimum set of observations is the one causing distortions under  $5 \mu\text{Gal}$ .

## 6 Configuration of the optimum set of observations

One of the five proposed sets of observations satisfies as the optimal design of the network measurement due to its small total flight distances, its high accuracy in estimating the unknowns, uniformly distribution of the absolute gravity accuracy across the network, and the minimal unavoidable distortions caused in the network. In this design, gravity difference measurements are to be done using relative gravimeter CG-3M in the forward and backward mode. The gravimeter is to be carried by an airplane between the stations. Each forward and backward measurement adds a new unknown parameter the instrumental drift rate to the number of unknowns. It is assumed that the measurement accuracy depends on the travel time for gravity difference measurements. There are totally 66 forth and back flights in this design. The longest flight line is 800 km and the smallest one is 100 km. Measurements of long distances such as Tehran to Tabriz, Mashhad, Isfahan, Shiraz, Yazd, Kerman, Bandar Abbas, Ahvaz and Kermanshah are proposed to be done in a shorter time interval using the Iran-Air commercial flights, while short distances are supposed to be done using the NCC charter planes usually used for the photogrammetry flights. To improve the accuracy of drift estimates, gravity is to be measured after arrival and before departure in every station. This will result in one degree of freedom for each drift. The gravity difference measurements of the selected design are classified in three distinct groups:

- a. Gravity difference measurements between the reference stations: there are totally 12 back and forth gravity difference measurements between the known reference stations. Using these measurements, the scale factor of the instrument is determined. In addition, a test on the instrument can also be conducted to see if the accuracy of the gravity difference measurement depends at least partially on the travel time of measurement by the gravimeter. For this, a model of variance accommodating for a linear dependency on the time of measurement was assumed as

$$\sigma_{\delta g}^2 = 50 + a\Delta t, \quad (19)$$

where 50 micro Gal squared is the two times of the variance of one gravimetry record,  $a$  is the coefficient to be determined, and  $\Delta t$  is the measurement time.

- b. Gravity difference measurements between the known and unknown stations: gravity difference measurements connecting each unknown station to at least three known stations were considered.
- c. Gravity difference measurements between all unknown stations: there are totally 15 measurements between unknown stations.

Table 2 summarizes the pre-analysis results of the measurement design.

## 7 Conclusions and discussions

The gravity base network proposed for Iran consisting of 19 stations uniformly covers the country. With the geometry of proposed observations considered, the maximum virtual distortion anticipated in the absolute gravity values at the network stations is less than  $5 \mu\text{Gal}$ . Measurement errors larger than  $25 \mu\text{Gal}$  are designed to be detected when entering the network, while the distortions caused by undetected errors (less than  $25 \mu\text{Gal}$ ) at the stations will remain smaller than  $5 \mu\text{Gal}$ . However, it is probable (10%) that some of the network stations may be deteriorated in accuracy by distortions larger than  $5 \mu\text{Gal}$ . To avoid such circumstances, safety measures are to be taken, e.g., gravimeter should be carried very carefully between stations, absolute and gravity difference measurements should be carried out in appropriate weather conditions. Magnitudes of the distortions estimated by Eq. (18) are surely datum dependent, i.e., they depend on the position(s) of reference station(s) relative to the other stations. Any change in the position of a reference station would affect the magnitudes. Since in the proposed design, the gravity datum is introduced by a set of 9 reference stations that uniformly cover the network and each unknown station is connected by at least 3 gravity difference measurements to 3 of the reference stations, the influence of the gravity datum (reference stations distribution) has been minimized.

Contrary to the 2-D horizontal geodetic networks in which the horizontal angles and distances are the measurable quantities, it is gravity that has to be measured in a gravity network. In a geodetic network, designing equal angles ( $60^\circ$ ) and equal distances may bring about a uniform distribution of stations and as such a strong network resisting the horizontal angles and distances errors. Designing a strong gravity network to resist the gravity measurement errors, i.e., a network not allowing measurement errors to evolve into the gravity distortions at the stations, however, requires stations in equal time (travel time) distances a part and the gravity differences of adjacent stations not abruptly varying in the neighborhood. This resembles a geodetic horizontal network of evenly distributed stations across. For this, at least the elevation differences (creating gravity differences) between the stations are reduced by avoiding the rough topography to be selected as a station location. Airports located in flat areas, escaping high lands, are suitable places in this respect. Also selecting airports as the stations would bring shorter and almost uniform travel time distances among the stations.

Tab. 2: Accuracy analysis of the network design

From	To	$d_i$	Internal reliability ( $\mu\text{Gal}$ )	gravity distortion at station #																		
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3	5	0.78	18.34	1	0	-3	0	1	0	1	0	0	0	0	0	1	1	0	-1	1	0	
3	6	0.71	19.23	0	1	-5	0	0	1	1	1	0	1	1	0	1	0	1	1	-1	1	0
17	6	0.87	17.37	0	1	-3	0	0	1	0	0	0	1	1	0	1	-1	0	1	-2	1	0
1	11	0.81	18.00	-1	1	-2	1	0	0	-1	-1	0	1	2	0	1	-1	1	0	-1	0	0
1	2	0.81	18.00	-2	2	-1	1	0	0	-1	-1	0	0	1	0	1	1	1	0	0	0	0
1	14	0.72	19.09	-2	1	0	1	1	-1	-1	-1	0	1	0	1	1	4	1	0	1	0	0
3	5	0.78	18.34	1	0	-3	0	1	0	1	0	0	0	0	0	1	1	0	-1	1	0	0
3	6	0.71	19.23	0	1	-5	0	0	1	1	1	0	1	1	0	1	0	1	1	-1	1	0
17	6	0.87	17.37	0	1	-3	0	0	1	0	0	0	1	1	0	1	-1	0	1	-2	1	0
1	11	0.81	18.00	-1	1	-2	1	0	0	-1	-1	0	1	2	0	1	-1	1	0	-1	0	0
1	2	0.81	18.00	-2	2	-1	1	0	0	-1	-1	0	0	1	0	1	1	1	0	0	0	0
1	14	0.72	19.09	-2	1	0	1	1	-1	-1	-1	0	1	0	1	1	4	1	0	1	0	0
1	15	0.83	17.78	-2	1	0	1	0	0	-1	-1	0	1	1	1	1	2	1	0	1	0	0
1	10	0.85	17.57	-2	0	1	0	0	0	0	0	1	1	1	1	0	1	1	0	1	0	0
1	12	0.84	17.68	-2	0	1	0	0	0	1	1	1	0	0	1	0	0	0	0	0	1	1
1	8	0.75	18.71	-2	-1	1	-1	0	0	3	3	1	0	0	0	-1	-1	0	1	0	1	1
1	7	0.70	19.36	-1	-1	0	-2	0	1	4	3	0	0	0	0	-1	-1	0	1	0	1	1
2	15	0.92	16.89	0	-1	0	-2	1	0	3	2	0	0	0	0	-1	0	1	0	0	1	0
3	2	0.94	16.71	0	0	-1	-3	1	0	2	1	0	0	0	0	0	-1	0	0	0	1	0
4	3	0.72	19.09	0	0	1	-4	1	0	1	0	0	0	0	0	0	-1	0	-1	-1	0	0
4	2	0.72	19.09	0	1	0	-5	1	0	0	0	0	0	0	0	0	-1	0	-1	-1	0	0
4	5	0.73	18.96	0	0	0	-5	0	0	0	0	0	0	0	0	0	1	0	-1	-1	0	0
4	6	0.77	18.46	0	0	0	-4	-1	0	0	0	0	0	0	0	0	1	0	-1	-1	0	0
4	6	0.77	18.46	0	0	0	-4	-1	0	0	0	0	0	0	0	0	1	0	-1	-1	0	0
5	2	0.81	18.00	0	1	0	-2	-3	0	-1	0	0	0	0	0	0	1	0	-1	0	0	0
5	14	0.69	19.50	1	0	-1	-1	-3	0	0	0	0	0	0	1	1	3	1	-1	0	0	0
5	15	0.75	18.71	1	0	-1	-1	-4	0	0	0	0	0	1	1	0	1	1	-1	0	0	0
5	1	0.78	18.34	0	0	-1	0	-4	-1	1	0	0	0	1	0	0	0	-1	0	1	0	0
5	7	0.73	18.96	0	1	-1	0	-3	-1	2	0	0	0	1	0	1	-1	0	0	0	1	0
6	2	0.77	18.46	0	2	-1	0	-1	-2	1	-1	0	0	1	1	1	0	1	0	0	1	0
6	5	0.82	17.89	1	1	-1	0	0	-3	1	0	0	1	1	1	1	1	1	0	0	1	0
6	1	0.80	18.11	1	1	-1	0	1	-3	1	1	1	1	1	1	1	1	1	0	0	1	1
6	7	0.80	18.11	1	0	-1	0	1	-2	1	2	0	1	1	0	1	1	0	0	0	1	1
6	8	0.72	19.09	0	0	-1	0	0	-2	-1	3	1	1	1	0	0	0	0	-1	1	1	1
7	10	0.75	18.71	0	0	-1	0	-1	-1	-4	3	0	1	1	0	0	0	0	-1	1	0	0
7	8	0.62	20.57	0	0	0	0	-1	-1	-4	4	-1	1	0	0	0	0	0	0	0	0	0
9	8	0.68	19.65	1	0	0	0	0	0	-1	4	-3	0	0	1	0	0	0	1	0	0	-1
9	12	0.72	19.09	1	1	1	1	1	1	0	2	-3	0	2	2	1	1	1	1	1	0	-1
9	11	0.70	19.36	1	0	1	1	1	0	0	0	-3	-1	3	3	1	1	1	1	1	0	-1
10	12	0.74	18.83	1	0	1	0	1	0	0	0	-1	-2	3	3	1	1	1	1	1	0	-1
10	11	0.73	18.96	0	0	0	0	0	0	0	1	1	-2	3	2	1	0	1	0	1	1	-1
10	9	0.75	18.71	0	0	0	0	0	0	0	1	1	-2	3	2	1	0	1	0	1	1	-1

Tab. 2: (Continue)

From	To	$d_i$	Internal reliability ( $\mu\text{Gal}$ )	gravity distortion at station #																		
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
10	8	0.70	19.36	0	1	0	1	0	1	1	3	1	-3	0	-1	0	1	-1	1	1	1	-1
10	2	0.80	18.11	0	1	0	1	0	0	0	2	0	-2	0	-2	1	2	-1	1	1	0	-1
12	14	0.64	20.25	0	0	1	1	1	0	1	2	-1	-1	0	-4	1	4	-1	1	1	0	-1
12	13	0.73	18.96	0	0	1	0	0	0	1	1	-1	0	2	-5	1	1	-1	0	1	0	0
12	11	0.59	21.09	0	0	1	0	0	0	1	1	0	1	4	-4	-1	-1	0	0	0	1	0
13	11	0.67	19.79	0	0	1	0	0	0	1	0	0	1	4	-2	-3	0	1	0	0	1	0
13	15	0.75	18.71	1	0	1	1	1	1	1	0	0	1	2	-1	-4	1	1	0	1	1	0
13	14	0.63	20.41	1	1	1	1	1	1	1	1	0	0	0	0	-4	3	0	1	1	0	0
13	2	0.68	19.65	0	2	0	1	0	1	0	0	0	0	-1	0	-4	0	-1	1	1	0	0
14	2	0.76	18.58	-1	3	0	1	0	0	0	0	0	0	0	-1	-3	-2	-2	0	1	0	0
15	14	0.70	19.36	0	1	0	0	0	0	1	1	0	0	-1	0	-2	2	-4	1	1	0	0
15	10	0.74	18.83	0	0	0	0	-1	0	1	1	0	1	0	0	-1	1	-4	0	0	1	0
15	12	0.75	18.71	0	0	0	-1	-1	0	1	0	1	1	1	1	-1	1	-4	0	0	1	0
15	11	0.72	19.09	0	0	0	-1	-1	0	1	0	1	1	2	1	0	0	-3	-2	0	1	1
16	6	0.83	17.78	0	0	0	0	0	0	0	0	1	1	1	1	0	0	-1	-3	1	0	0
16	4	0.77	18.46	0	1	0	1	0	0	0	0	0	1	1	1	1	1	0	-3	1	0	0
16	5	0.76	18.58	1	1	0	1	1	0	1	0	0	0	0	1	0	1	1	-4	1	0	0
16	8	0.77	18.46	1	0	0	0	1	1	1	2	0	0	0	0	0	0	1	-3	0	1	0
16	7	0.78	18.34	0	0	1	1	0	1	2	1	0	0	0	0	0	0	0	-2	-2	1	0
17	4	0.77	18.46	0	0	1	0	0	0	1	1	0	0	-1	0	0	-1	0	-1	-4	1	0
17	3	0.65	20.09	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	0	0	-5	0	0
17	2	0.74	18.83	0	0	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	-5	-1	0
17	14	0.68	19.65	0	-1	0	0	1	0	-1	-1	0	0	0	1	0	3	0	0	-4	-2	0
18	11	0.77	18.46	0	0	0	0	0	-1	-1	-1	0	0	1	1	0	1	0	0	-2	-3	0
18	9	0.74	18.83	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	-1	-4	0
18	8	0.73	18.96	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	-4	-1
18	7	0.71	19.23	0	0	0	1	0	1	2	1	-1	0	0	0	0	0	0	1	1	-4	-2
18	10	0.83	17.78	0	0	0	1	0	0	1	1	0	1	0	0	0	0	0	0	1	-2	-3
19	9	0.69	19.50	1	0	0	1	0	1	1	1	1	1	1	1	0	0	1	1	1	0	-5
19	18	0.70	19.36	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	-5
19	10	0.80	18.11	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	-3
1	1	0.83	10.67	2	1	1	1	2	1	2	1	1	1	1	2	1	2	2	1	1	1	1
2	2	0.68	11.79	1	4	1	2	2	1	1	0	0	1	0	1	2	0	2	1	2	0	1
3	3	0.60	12.55	1	1	5	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1
6	6	0.81	10.80	1	1	1	1	1	2	2	2	1	1	1	1	1	1	1	1	1	1	1
9	9	0.64	12.15	1	1	1	1	1	1	0	2	4	1	2	2	1	1	1	1	1	2	2
10	10	0.82	10.73	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	1	1	1	1
13	13	0.76	11.15	1	2	1	1	1	1	1	0	1	1	2	1	3	2	1	1	2	1	1
16	16	0.78	11.01	1	1	1	1	1	1	2	2	1	1	1	1	1	1	2	1	2	1	1
19	19	0.67	11.87	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	4

Tab. 2: (Continue)

Standard deviations of gravity stations			
Standard deviation	Station Number	Standard deviation	Station Number
1.24	1	1.92	11
1.70	2	1.71	12
1.91	3	1.47	13
1.46	4	2.77	14
1.59	5	1.58	15
1.31	6	1.42	16
2.15	7	1.67	17
1.83	8	1.59	18
1.79	9	1.74	19
1.26	10		

A linear drift model was assumed for the CG-3M gravimeter in the observation equation (3). This would require the measurements to be carried out as fast as possible between the stations. For this, it is proposed that the long distances to be measured using national flights. Short connections can be measured using the charters of the National Cartographic Center (NCC). Finally, it can be said that the designed observations for the gravity base network will bring about accuracies of better than  $5 \mu\text{Gal}$ . The parallel use of second gravimeter side by side of the first gravimeter in the network measurement would bring about more redundancy of the observations even if one more scale factor and some drift rates are added to the number of nuisance parameters to be solved for in the network.

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## Abstract

A gravity base network including 19 stations is proposed for Iran. For the fast accessibility by a gravimeter, the stations are assumed in the national airports. Nine of the stations are selected as the reference stations at which absolute gravity values are to be measured. Six of them make a calibration line extending from the extreme North-West to the extreme South-East end of the country. The remaining stations, uniformly distributed throughout, are connected to at least 3 reference stations by precise gravity difference measurements. Based on the  $5 \mu\text{Gal}$  accuracy of the CG-3M gravimeter and the assumed  $3 \mu\text{Gal}$  accuracy of FG5 absolute gravimeter, pre-analysis of five different designs of the network observations was carried out. The reference stations were considered initially known at absolute gravity values with observational weights inversely proportional to the variance of absolute measurements. The drift of gravimeter was assumed linear in time within an interval of 8 hours. Individual gravimeter readings were assumed

uncorrelated, while mathematical correlations among reading differences (gravity difference measurements) were assessed. Gravity difference measurement accuracies were taken to be partly proportional to the spent time of measurements with the constant of proportionality to be determined in the computation. Redundancy numbers and internal reliabilities at 90 % confidence level were evaluated to study the robustness of the network against gross errors. Individual undetectable gross errors were then applied to show the corresponding distortions in the network. In the optimal design, the redundancy numbers, all larger than 0.5, are uniformly distributed throughout the network and the achievable accuracy is the best. Even for the optimal design, gross errors up to  $25 \mu\text{Gal}$  may survive the statistical test, but the distortion caused by these errors would remain under the sought accuracy of  $5 \mu\text{Gal}$ .

## Keywords

Gravity Base network, gravimeter, gravimetry, network design, internal reliability.