Parameter Estimation by Means of Genetic Algorithms: Potential and Limitations from a Geodetic Perspective

Parameterschätzung mittels genetischer Algorithmen: Potenzial und Grenzen aus geodätischer Perspektive

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In the last decades, optimization problems in geodesy have been becoming increasingly numerous and complex. Especially for highly non-linear problems, (gradient-based) local search methods often fail to detect the optimal solution, i.e., the global minimum or maximum of the solution space. By mimicking biological evolutionary processes in terms of mathematical operations, genetic algorithms are able to abandon local optima in favor of the global optimum; as such, they can be tailored to parameter estimation purposes. In this context, this contribution assesses the performance of genetic algorithms in inverse modeling, net adjustment, time series analysis and orbit design. From the numerical experiments it turns out that the solution quality is strongly dependent on the optimization problem and the proper choice of steering parameters such as search domain bounds, population size, selection rule and mating principle. As a general conclusion, the combination of truncated genetic algorithms with a local optimizer is highly recommendable for practical applications.

Keywords: Global optimization, inverse modeling, net adjustment, time series analysis, orbit design

Während der letzten Jahrzehnte hat die Anzahl und Komplexität von Optimierungsproblemen stark zugenommen. Diese Entwicklung macht auch vor der Geodäsie nicht halt. Dabei gelingt es vor allem bei hoch nicht-linearen Problemen oftmals nicht, mithilfe von (Gradienten-basierten) lokalen Suchverfahren die optimale Lösung, sprich das globale Minimum oder Maximum, ausfindig zu machen. Genetische Algorithmen können hier Abhilfe schaffen. Durch das Abbilden biologischer Evolutionsprozesse in mathematische Operatoren sind sie in der Lage, lokale Optima zugunsten des globalen Optimums zu verlassen. Infolgedessen kann das globale Optimierungsverfahren für die Parameterschätzung angepasst werden. In diesem Kontext beschreibt und bewertet dieser Beitrag die Leistungsfähigkeit genetischer Algorithmen für die inverse Modellierung, Netzausgleichung, Zeitreihenanalyse und das Bahndesign. Die numerischen Studien zeigen, dass die Qualität der Lösungen einerseits sehr stark von der Art des Optimierungsproblems abhängt. Andererseits nimmt die Wahl der Strategieparameter, wie beispielsweise Suchraumschranken, Populationsgröße, Selektionsregel und Mutationsprinzip, einen großen Einfluss auf die Lösungsfindung. Als allgemeine Aussage läßt sich festhalten, daß für praktische Anwendungen die Kombination aus frühzeitig abgebrochenem genetischem Algorithmus mit anschließender lokaler Optimierung eine vielversprechende Strategie darstellt.

Schlüsselwörter: Globale Optimierung, inverse Modellierung, Netzausgleichung, Zeitreihenanalyse, Bahndesign

1 INTRODUCTION

Optimization problems may be considered as "the staff of life" for a wide range of engineering disciplines. Extremum problems according to $f(\boldsymbol{x}) = f(x_1,\ldots,x_u) = \min_{\boldsymbol{x}}!$ constitute the classical category of optimization problems [for the sake of simplicity, here we formulate any optimization problem as minimization problem, keeping in mind that maximization turns into minimization by a change of algebraic sign]. In case of linear problems, $f(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x}$, the adoption of local search algorithms yields the optimal solution. This is owing to the fact that for linear problems, the (single) local minimum is equal to the global minimum. As a consequence, both approaches based on function values and function derivatives are suitable, although the speed of convergence can vary significantly. Amongst others, the former include bisection (e.g., /Burden, Faires 2011/) and the downhill simplex method /Nelder, Mead 1965/. Techniques based on function derivatives include Newton's method and (conjugate) gradient methods /Hestenes, Stiefel 1952/.

For non-linear problems, local optimizers potentially fail to find the global minimum; the goodness of the result depends on the starting point of the local search (*Fig. 1*). As a consequence, to end up at the optimal solution requires the starting point (also referred to as approximate values) to be "close enough" to the global minimum. As a matter of fact, proper approximate values are often unavailable in practice. This is a serious concern; especially for highly non-linear problems, where small variations in the parameter space have a large impact in the solution space.

In the last decades, a considerable number of global optimization strategies has been proposed to overcome the limitations of local search algorithms. Amongst them, stochastic (for instance, Monte-Carlo simulation based) methods and evolutionary computation gained particular attention in engineering. Established representatives of the former include simulated annealing /Kirkpatrick et al. 1983/ and stochastic tunneling /Wenzel, Hamacher 1999/. Evolutionary computation, on the other hand, comprises techniques based on swarm intelligence (e.g., ant colony algorithms; /Dorigo, Stützle 2004/) and evolutionary algorithms such as evolutionary strategies /Rechenberg 1994/ and genetic algorithms /Holland 1993/. Importantly, global optimization methods share the philosophy to abandon local minima in favor of the global minimum (*Fig. 1*).

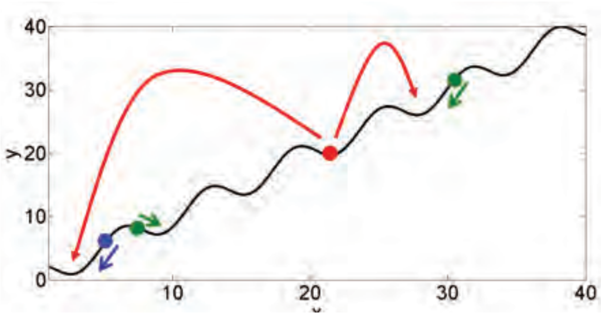

Fig. 1 | Local versus global search. Local optimizers yield the optimal solution if and only if the starting point is close enough to the global minimum (blue); otherwise, a local minimum is reached (green). Global optimizers allow escaping from a local minimum in favor of other local minima and the global minimum (red).

This contribution sheds light on the performance of genetic algorithms (GA) for parameter estimation purposes from an application point of view. Examples are taken from different geodetic disciplines, namely inverse modeling, net adjustment, time series analysis and orbit design. The GA coding used here is an adaption of the source code by /Haupt, Haupt 2004/. Apart from the straightforward GA implementation, the work by /Haupt, Haupt 2004/ is highly recommendable from an educational perspective. In order to avoid own coding, the MATLAB gatool may be used, which is distributed within the MATLAB Global Optimization Toolbox /MathWorks 2012/.

2 PROBLEM FORMULATION

In geodesy, optimization problems occur whenever a set of unknown model parameters, $\boldsymbol{x}\ [u \times 1]$,is sought for from a series of observations $\boldsymbol{y}\ [n \times 1]$.Typically the number of observations exceeds the number of model parameters ($n > u$), yielding an over-determined (and inconsistent) system of equations $f(\boldsymbol{x}) = \boldsymbol{y} + \boldsymbol{e}$, with f describing the functional relation between $\boldsymbol{x}$ and $\boldsymbol{y}$; $\boldsymbol{e}\ [n \times 1]$ denotes the vector of inconsistencies. In an L_2-norm minimization sense, any problem of this kind yields the cost function

$$F(\boldsymbol{x}) = \boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} = [f(\boldsymbol{x}) - \boldsymbol{y}]^{\mathrm{T}}[f(\boldsymbol{x}) - \boldsymbol{y}] = \sum_{i=1}^{n}[f(x_1,\ldots,x_u) - y_i]^2 = \min_{\boldsymbol{x}}. \tag{1}$$

Linear models, $f(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x}$, turn Eq. (1) into a linear least-squares (LS) problem. Its solution ist he best linear unbiased estimator $\hat{\boldsymbol{x}} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{y}$ /Koch 1999/. LS adjustment can also be applied to non-linear models by linearization of the functional relation, $f(\boldsymbol{x}) \approx f(\boldsymbol{x}_0) + \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}}|_{\boldsymbol{x}_0}\ (\boldsymbol{x} - \boldsymbol{x}_0)$, in combination with iterative parameter estimation. This procedure, however, requires that (i) the linearization of the functional relation is possible and (ii) proper approximate values $\boldsymbol{x}_0$ are available. If (at least) one of these conditions is not fulfilled, LS adjustment will fail to provide the optimal solution. In this case, global optimization is a more appropriate parameter estimation tool.

3 GENETIC ALGORITHMS

Basically, GA iteratively find the best solution to an optimization problem starting from random samples. Therefore, no a priori information is necessary. Importantly, GA are working on functional values directly, rather than taking partial derivatives, i.e., they avoid linearization. The cost function (also referred to as fitness function or target function) to be optimized can be defined in several ways. In order to maintain consistency with LS adjustment, in the sequel Eq. (1) is defined as cost function.

The GA principle is mimicking biological evolutionary processes in terms of mathematical operations (e.g., /Goldberg 1989/, /Holland 1993/). Therefore, most of the terminology is taken from biology. GA either represent variables as encoded binary strings or work with the continuous values themselves. Both the binary and continuous representations are equivalent. Binary strings, however, are more didactical. Converting continuous into binary values, and vice versa, is performed by appropriate encoding and decoding rules /Haupt, Haupt 2004/.

Fig. 2 outlines the GA procedure. In a first step, the GA parameters are defined and a set of random "solution vectors" is generated. One particular random sample vector represents an individual; the whole set is known as initial population. Furthermore, the initialization specifies the lower and upper search domain bounds for each individual parameter to be optimized. The more restricted the domains the faster the algorithm converges.

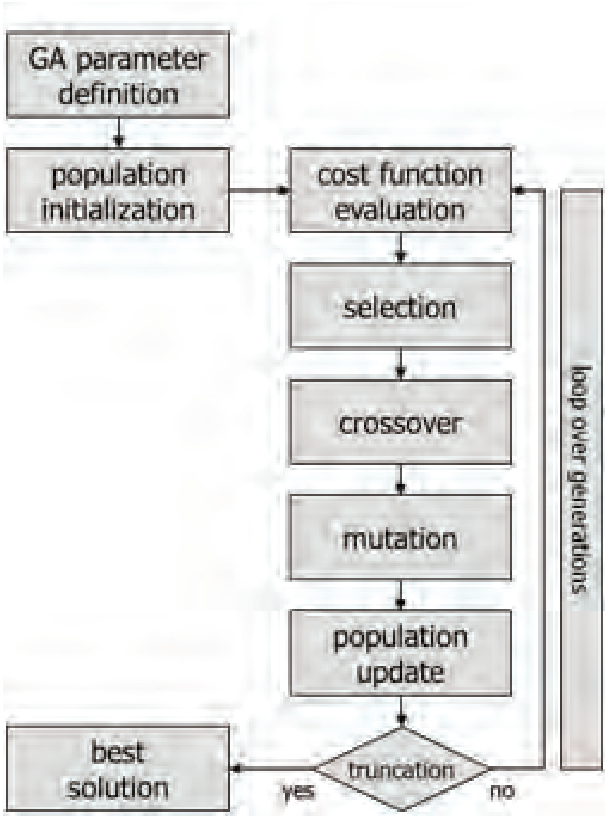

Fig. 2 | Genetic algorithm principle. Based on an initial random population, the algorithm searches for the best solution (lowest cost) by successively applying the operators selection, crossover and mutation.

After initialization, the iterative optimization process starts. Within each iteration, cost function evaluation provides the "fitness" of each individual. In order to generate an optimal solution in terms of lowest cost, some (or even all) individuals of the population have to change. Hence, the population evolves over time. For this reason, the iterations produce successive generations of individuals. The iterative process itself is a repeated sequence of (genetic) operators applied to the actual generation.

The first of the GA operators is selection. It specifies the individuals that survive to the next generation (expressed as keep rate) and the number of individuals that will be replaced by offspring children). A keep rate of 50%, for instance, implies that the fitter half of the individuals survives. The crossover operator "produces" the offspring by combining the genetic information of parent individuals. Practically, parts of the binary representation of parent individuals are composed in order to form new individuals. In literature, a huge variety of selection rules and mating principles are proposed /Haupt, Haupt 2004/. We applied cost-proportional selection and single-point crossover. The latter combines genetic information of two (randomly selected) parents considering a single (randomly selected) crossover point (*cf. Tab. 1*).

The final operator is mutation. It operates at the bit copy level; when the bits are being copied from the parent to the offspring, each bit may become mutated according to a pre-defined probability, cf. the last two columns of *Tab. 1*. From the methodological point of view, mutation is important to guarantee that the algorithm will not get stuck at a local minimum. Survived parents and their offspring define the individuals of the new generation; they replace the previous population.

The iterative procedure terminates once an a priori defined truncation criterion is reached. Amongst others, truncation may be linked to a fixed number of iterations, a lower bound of the minimal cost or differences between successive "best" solutions or minimal costs. In any case, under the postulate of the "survival of the fittest", GA provide the optimal solution according to the environmental conditions.

However, GA typically show slow convergence, and hence parameter estimation potentially becomes a very time-consuming task. In order to speed up the procedure, GA should be complemented with a local optimizer once the GA solution is close to the global minimum (*Fig. 3*). Following this two-step principle, the GA solution serves as starting values for local search, which is usually considerably less time-consuming compared to the repeatedly population-wide cost function evaluation within the global optimization. Hence, GA may be interpreted as a tool to find approximate values that ensure local search algorithms to find the global optimum. However, the choice of the GA truncation point is a critical issue, as there is no guarantee that the truncation criteria mentioned above indeed provide a solution close enough to the global minimum. In a statistical sense, mul-

String No.	Mating pool (potential parents)	Mate (randomly selected)	Crossover point (randomly selected)	Offspring after mating	Mutated bits (randomly selected)	Offspring after mutation
1	0 1 1 0 1	2	4	0 1 1 0 0	0	0 1 1 0 0
2	1 1 1 0 0	1	4	1 1 1 0 1	1,3	0 1 0 0 1
3	1 1 0 0 0	4	2	1 1 0 1 1	1	0 1 0 1 1
4	1 0 0 1 1	3	2	1 0 0 0 0	5	1 0 0 0 1

Tab. 1 | Exemplary single-point crossover and offspring mutation in binary representation tiple evaluations (e.g. 50 independently performed computations) can help to assure that, at least for some of these computations, the GA result is indeed in appropriate vicinity to the global minimum. Although this approach requires additional computation time, for many optimization problems (especially when a huge number of parameters has to be estimated and/or the solution space is huge) the procedure is still more effective compared to stand-alone GA.

The local optimizer used in this study is the downhill simplex method (DSM; /Melder, Nead 1965/). This choice is representative for local optimization methods operating on the level of function values (opposed to gradient methods). The term "simplex" describes a polytope with (d+1) vertices in the d-dimensional space; this (irregular) geometrical body is a generalization of the triangle (d=2) or the tetrahedron (d=3) to arbitrary dimension. By successive reflection, expansion and contraction of the simplex, it "tumbles downhill" the d-dimensional topography. The DSM algorithm stops if one of the simplex vertices encounters the minimum (lowest cost). Please consult /Press et al. 1992/ for more details on the DSM method and its implementation.

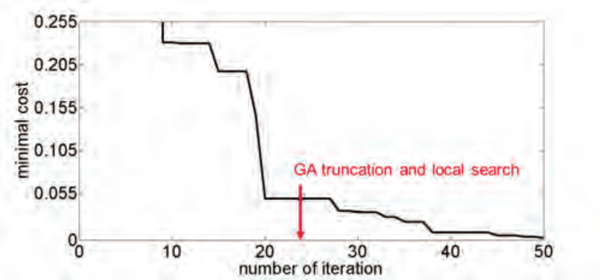

Fig. 3 | Typical GA convergence behavior. The algorithm requires a considerable number of iterations (black line) once the solution is already close to the global minimum. Truncation of the GA (indicated in red) in combination with subsequent local optimization saves computation time.

4 APPLICATIONS IN GEODESY

The application of GA in geodesy has not been emphasized prior to the turn of the millennium. In the last decade, however, several branches of geodetic science started to adopt global optimization for their purposes. This is mainly for two reasons. Firstly, the steadily increasing complexity of geodetic optimization problems asks for such a development. Secondly, nowadays the comprehensive access to high performance computing platforms makes runtime to become a less critical limiting factor. GPS attitude determination /Xu et al. 2002/, quasi-conformal mapping /González-Matesanz, Malpica 2006/, local gravity field modeling /Antoni 2012/ and surveying network design /Rehr et al. 2011/, /Saleh, Chelouah 2004/ are only a few examples that demonstrate the potential of GA in geodesy. The following subsections provide a closer look on inverse modeling, net adjustment, time series analysis and orbit design.

4.1 Inverse modeling

The inference of mass anomalies in the Earth's interior from gravity measurements on the Earth surface is one of the oldest and most challenging optimization problems in geodesy. Accordingly, opposed to other geodetic disciplines GA have some "tradition" in inverse modeling /Alvers 1998/, /Montesinos et al. 2005/, /Chen et al. 2006/. No unique solution to the inverse modeling problem exists, which is due to the fact that different buried anomalies cause similar, even identical, surface signals. Consequently, constraints on the solution are indispensable.

Schematically, *Fig. 4* (left panel) shows simulated surface gravity anomalies (contaminated by normally distributed errors with a standard deviation of 20µGal) as caused by a buried mass anomaly of cylindrical shape.In order to infer the anomaly geometry from the observed surface signal, GA constraints in terms of form parameters were considered; these form parameters are depth b, cylinder base area radius a and cylinder length l. Hence, according to Eq. (1) the cost function becomes

$$F(a,b,l)=\sum_{i=1}^{n}\left[\Delta g_i(a,b,l)-\Delta g_i^{obs}\right]^2=\min_{a,b,l}. \qquad (2)$$

In Eq. (2), Δg_i^{obs} denote observed gravity anomalies; $\Delta g_i(a,b,l)$ are forward-evaluated gravity anomalies /Tsoulis 1999/ dependent on the form parameters to be optimized. With search domain bounds for each individual parameter of $\pm 10\%$ of the input values, the GA recovered the form parameters with an accuracy of much less than 1% /Körner 2010/. However, this result has to be taken with care. Due to the problem ambiguity, the quality of the estimated parameters highly depends on the search domain bounds, i.e., on a priori problem knowledge.

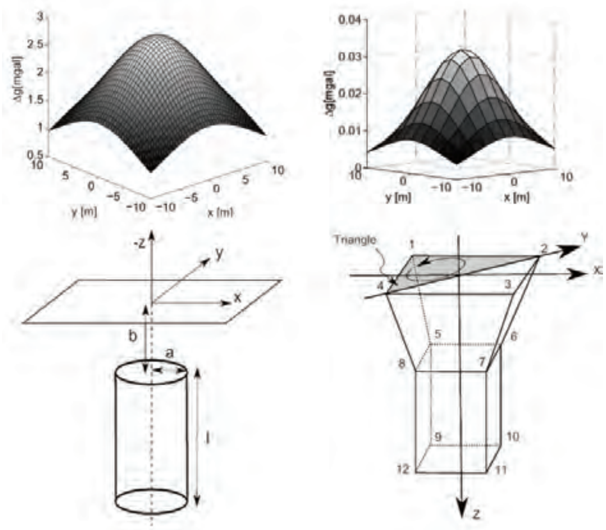

Fig. 4 | Inference of buried mass anomalies from observed surface gravity signal. Left: the cylindrical mass anomaly is represented in terms of the form parameters a, b and l; right: the 3D coordinates of the polyhedron vertices 1 to 12 constitute the optimization parameters (taken from /Körner 2010/).

Form parameters constitute strong constraints as they assume knowledge about the geometrical shape (and depth) of the buried body. More general, an arbitrary 3D structure can be modeled in terms of a polyhedron /Tsoulis 1999/; in that case, the coordinates $x_j, y_j, z_j, j=1,\ldots,k$ of the polyhedron vertices – assembled in the vector $\boldsymbol{x}$ – are sought for (*Fig. 4*, right panel), and hence the cost function reads

$$F(\boldsymbol{x}) = \sum_{i=1}^{n} \left[\Delta g_i(x_1,\ldots,x_k,y_1,\ldots,y_k,z_1,\ldots,z_k) - \Delta g_i^{\text{obs}}\right]^2 = \min_{\boldsymbol{x}}. \quad (3)$$

Within the optimization process, constraints are introduced by lower and upper search domain bounds for each individual coordinate. From numerical studies /Körner 2010/, again it turned out that the quality of the results is highly subject to the allowed parameter range, which is due to the nature of inverse problems. Furthermore, both the choice of GA parameters (population size, mutation rate, selection rule, etc.) and the overall number of optimization parameters have a severe impact on runtime. In summary, in inverse modeling the performance of GA is heavily driven by problem-specific knowledge and the trial-and-error-based finding of proper GA parameters.

4.2 Net adjustment

In surveying, the coordinates of points are typically derived from distance and angle measurements using geometric methods like intersection, arc section, traverse, etc. Here we have a closer look on free net adjustment. In this context, special attention is paid on the consideration of observation weights and restrictions within the GA-based parameter estimation process. Whereas the former is particularly essential when dealing with different observation types (for instance, distances and angles), the introduction of restrictions allows solving the datum problem, which is inherent to a free net configuration. The GA cost function for weighted LS with restrictions is

$$F(\boldsymbol{x}) = \sum_{i=1}^{n} \left\{ w_i^s \left[s_i(\boldsymbol{x}) - s_i^{\text{obs}}\right]^2 + w_i^\alpha \left[\alpha_i(\boldsymbol{x}) - \alpha_i^{\text{obs}}\right]^2 \right\} + \sum_j w_j^c \left[c_j(\boldsymbol{x})\right]^2 = \min_{\boldsymbol{x}}. \quad (4)$$

In Eq. (4) it is assumed that the net adjustment is according to observed distances s_i^{obs} and angles α_i^{obs} as outlined in *Fig. 5*. $s_i(\boldsymbol{x})$ and $\alpha_i(\boldsymbol{x})$ are forward-evaluated values dependent on the vector $\boldsymbol{x}$, which assembles the parameter to be optimized, i.e., the Cartesian coordinates of the network points. Observation weights – w_i^s and w_i^α in Eq. (4) – are considered by simply scaling the misfit between forward-modeled values and observed values with inverse a priori variance information. The higher the weight, the smaller the misfit of the corresponding individual term in Eq. (4) has to be in order to minimize the cost function. Accordingly, restrictions $c_j(\boldsymbol{x})$ on the parameter vector $\boldsymbol{x}$ are introduced by attributing very high (theoretically infinite high) weights, w_j^c, to those restrictions.

To demonstrate the feasibility of the proposed strategy, *Tab. 2* summarizes the results for constraint weighted LS applied to the free network in *Fig. 5* using GA and LS adjustment. The datum problem was solved by fixing the coordinates x_1, y_1 and x_2, i.e., $c_1(\boldsymbol{x}) = (x_1 - x_1^0)$, $c_2(\boldsymbol{x}) = (y_1 - y_1^0)$, $c_3(\boldsymbol{x}) = (x_2 - x_2^0)$ holds true; x_1^0, y_1^0 and x_2^0 denote fixed a priori values. The error information (standard deviations) in the second column of *Tab. 2* is based on the comparison of the GA result using the actual observations and the GA result using 'manipulated' observations; here, a'manipulated' value is the sum of an observation and its standard deviation. For LS adjustment, the standard deviations in *Tab. 2* (third column) were derived from the inverse normal equations matrix.

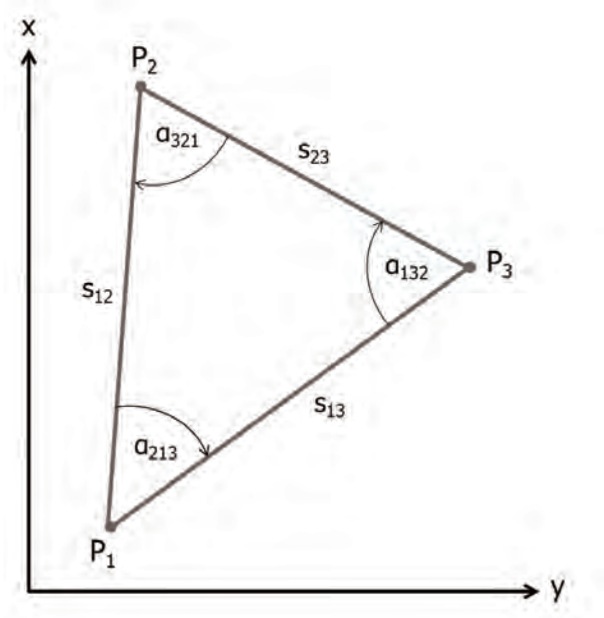

Fig. 5 | Free network adjustment configuration. The 2D coordinates of the points P_1 to P_3 shall be determined from three distance observations and three angle observations.

Parameter	GA	LS adjustment
y_2	132.496 ± 0.02	132.496 ± 0.02
x_3	98.893 ± 0.01	98.893 ± 0.02
y_3	59.827 ± 0.01	59.827 ± 0.01

Tab. 2 | Numerical results for constraint weighted LS applied to the free network in Fig. 5

Both GA and LS adjustment provide comparable results. This also holds for more complex (synthetic) 2D net configurations as well as for other geometric surveying methods (not shown here). From the experiments it turned out that the GA performance is largely independent from the choice of GA parameters;due to fast convergence, runtime is not an issue. Certainly, from the viewpoint of network complexity the 2D example above illustrates a rather simple problem configuration. As far as convergence is concerned, net adjustments characterized by stronger non-linearity (e.g., 3D networks) and a larger number of unknown parameters are probably harder to deal with. Furthermore, it has to be emphasized that – as opposed to least-squares adjustment – the full variance-covariance information of the estimated parameters is not directly accessible.

4.3 Time series analysis

Extracting oscillations and trends from sampled data is one of the most relevant challenges in (geodetic) time series analysis. Classically, Fourier analysis is used for this task. The technique fits signal amplitudes and phases to a number of sinusoids. Although the consideration of a huge number of sinusoids basically allows to capture the overall content of any signal, (geo-)physical interpretation of the periodicities has to be done with care. In order to avoid the signal energy to be forced into fixed periods, alternatively the frequencies can be treated as additional unknown parameters; this yields time series analysis to become a highly non-linear optimization problem. Summarizing the amplitudes, frequencies, phases and polynomial coefficients in the vectors $\boldsymbol{a}, \boldsymbol{f}, \boldsymbol{\varphi}$ and $\boldsymbol{p}$, respectively, results in the GA cost function

$$F(\boldsymbol{a}, \boldsymbol{f}, \boldsymbol{\varphi}, \boldsymbol{p}) = \sum_{i=1}^{n} \left[\left(\sum_{k=1}^{s} a_k \sin(2\pi f_k t_i + \varphi_k) + \sum_{j=0}^{m} p_j t_i^j \right) - y_i^{\text{obs}} \right]^2 = \min_{\boldsymbol{a}, \boldsymbol{f}, \boldsymbol{\varphi}, \boldsymbol{p}}. \quad (5)$$

Importantly, the estimation of $\boldsymbol{a}, \boldsymbol{\varphi}$ and $\boldsymbol{p}$ can be separated as linear problems from Eq. (5). That is, only the frequencies $\boldsymbol{f}$ remain to be estimated with the global optimizer. This is essential from the computational point of view, as the number of GA optimization parameters reduces significantly. For more details on the technique and its implementation we refer to the comprehensive work by /Mautz 2001/.

We applied GA-based frequency estimation to GRACE (Gravity Recovery And Climate Experiment) time series analysis /Schmidt et al. 2008/ and /Baur 2012/ for more information on mass-variation time series from GRACE). As a first experiment, a synthetic Greenland time series consisting of four sinusoids (annual signal, semi-annual signal, S2 tidal alias, K2 tidal alias) and a linear trend function has been modeled; noise was added to the synthetic series in order to more realistically replicate real data. Inspection of *Fig. 6* (upper panel) reveals that the noise-free input signal could be reconstructed successfully from the noisy data. Noteworthy, the frequency errors are well below 1%. In a second experiment, we analyzed a 9-years real-data time series of the Amazon basin (*Fig. 6*, lower panel). The major frequencies were detected to 363.6 days and 904.7 days. Both the annual signal and the 2.5-yearly oscillation result from hydro-meteorological patterns /Schmidt et al. 2008/.

From a series of additional experiments it turned out that the results are robust against the presence of data gaps and non-uniform sampling rate. The choice of the GA parameters is not too critical; however, the (quality and runtime) performance of the technique decreases with increasing frequencies to be optimized. As a consequence, the procedure has to be adapted to individual needs.

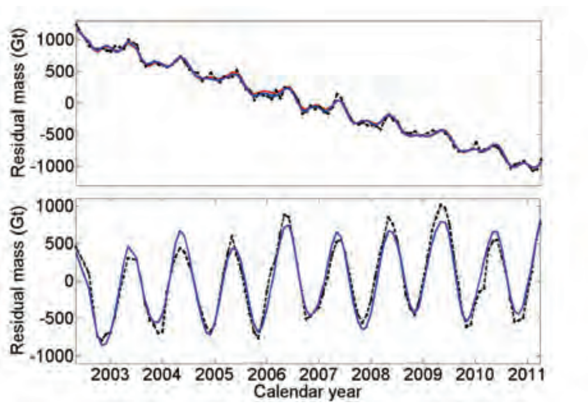

Fig. 6 | GRACE time series analysis. Upper panel: Greenland mass variation signal (synthetic); lower panel: Amazon basin mass variation signal (observed). Red line: synthetic noise-free signal; black lines: noisy signals; blue lines: reconstructed signals.

4.4 Orbit design

The design of satellites orbits according to pre-defined requirements is all but a trivial task. Long-term stability is particularly challenging, especially for low-altitude satellites orbiting celestial bodies with rough gravity field features /Ramanan, Adimurthy 2005/. In order to minimize the number of thruster events, numerous studies proposed GA to find a solution to the orbit design problem /Abdelkhalik, Gad 2011/, and the references therein). As the evolution of the orbit is mainly dependent on the initial satellite position $\boldsymbol{r}_0$ and velocity $\boldsymbol{v}_0$ (or initial Keplerian elements), the cost function can be formulated as

$$F(\boldsymbol{r}_0, \boldsymbol{v}_0) = \frac{\mathrm{d}p(\boldsymbol{r}_0, \boldsymbol{v}_0)}{\mathrm{d}t} = \min_{\boldsymbol{r}_0, \boldsymbol{v}_0}. \quad (6)$$

In Eq. (6), $p = a(1-e)$ denotes the (oscillating) periapsis distance; a and e are the (oscillating) orbit altitude and orbit eccentricity, respectively.

Fig. 7 presents the sensitivity of the orbit decay on the choice of the initial state vector. In this experiment, only the orbit inclination is a variable parameter; all other Keplerian elements have been held fixed. The orbit integration results show that within the initial inclination range from $i_0 = 83°$ to $i_0 = 87°$ the orbit evolution varies significantly. For $i_0 = 85°$ the orbit is nearly stable, whereas for $i_0 = 87°$ the satellite impacts the surface after some 300 days.

5 CONCLUSIONS

Genetic algorithms may be considered as a crude brute-force method for solving (non-linear) parameter estimation problems. Basically, the algorithm can be interpreted as structured successive forward modeling until the parameter vector adequately fulfills the cost func-

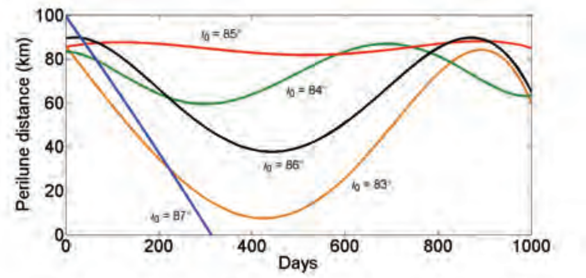

Fig. 7 | Decay of a Moon-orbiting satellite dependent on the initial orbit inclination. The results are based on orbit integration with the LP100J lunar gravity field model up to spherical harmonic degree and order 50.

tion. The idea to obtain the optimal global solution without any a priori problem knowledge is highly appealing – although there is no definite guarantee that the heuristic method indeed succeeds in detecting the global minimum/maximum. On the other hand, both the solution quality and runtime performance are highly problem-dependent; as a general statement, they decrease with increasing number of parameters to be optimized. In addition, the GA parameters (population size, selection rule, mating principle, etc.) should be chosen wisely; trial and error is laborious but helpful. GA are very well suited for parallel programming; from that point of view, runtime becomes a less critical issue. Furthermore, the combination of truncated GA with a local optimizer is highly recommendable for practical applications.

Compared to non-linear LS adjustment, GA circumvent linearization of the functional relation between the unknown parameters and the observations. Nevertheless, if gradient information is available, LS adjustment might be preferred over GA – simply because LS adjustment provides direct access to the variance-covariance matrix of the estimated parameters. In this case, GA are an appropriate tool to find high-quality approximate values to be used for the gradient-based local search.

The lower and upper search domain bounds for the parameters to be optimized are the most influential GA steering parameters. The more restricted the bounds are, the more a priori information goes into the algorithm, and hence the faster the GA converges. That is, problem knowledge reduces runtime, and vice versa. This behavior is in accordance with the "no free lunch theorem for search and optimization" /Wolpert, Macready 1997/.

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