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Fitting Free-Form Surfaces to Laserscan Data by NURBS

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Nicht nur im Maschinenbau, auch in der Ingenieurgeodäsie muss die Aufgabe gelöst werden, Freiformflächen an Daten von Laserscannern oder ähnlichen Instrumenten anzupassen. Dabei treten fast ebenso viele unbekannte Parameter wie Datenpunkte auf. Ein schnelles Verfahren zur Anpassung von Flächen mit NURBS (nonuniform rational B-splines) wird daher mit Hilfe der Schätzung sich überkreuzender Kurven angewendet. Es benötigt näherungsweise nur die Wurzel aus der Anzahl der Datenpunkte als unbekannte Parameter. Mit einfachen Beispielen generierter Messungen und Messungen des Laserscanners Leica HDS 3000 wird gezeigt, dass die Anpassung der Freiformflächen mit einer Genaugkeit erfolgen kann, die die Genauigkeit der gemessenen Daten approximiert.

Introduction

Objects can now be recorded with high resolution within a short time by laserscanners or similar instruments like lasertrackers. The aim of the data acquisition is often the analytical representation of the surface being scanned. This is not difficult for simple surfaces like planes, spheres or cylinders. More demanding are free-form surfaces. This task arises also for reverse engineering which is now discussed in engineering geodesy, cf. Hennes (2006), Herrmann and Möser (2008). In reverse engineering the coordinates of points at the surface of a manufactured object are measured and then approximated by an anlytical model of computer-aided design. This is accomplished by either interactive construction of the model or by fitting a surface to the data or by a combintion of both methods.

Reverse engineering generally uses NURBS (nonuniform rational B-splines) or their special cases, nonuniform nonrational B-splines, to represent surfaces. A vast literature on NURBS exists as well as on the special tasks of reverse engineering. There are books introducing NURBS like Piegl and Tiller (1997), Farin and Hansford (2000), Rogers (2001) and review articles in journals like Piegl (1991). For an introduction to reverse engineering see Varady et al. (1997). Software exists like Rhinoceros for constructing NURBS surfaces.

In the following we will solely concentrate on fitting free-form surfaces in three-dimensional space to measured data points. NURBS are used to represent the surfaces in a parametric form. The points of a NURBS surface are linearly related to a grid of unknown control points, if the knots of the B-spline basis functions, the location parameters of the measured points and the weights of the control points are known. The unknown control points can be determined by interpolating the measured points by a surface as shown by Barsky and Greenberg (1980). The linear relations may also be used as observation equations for a simultaneous estimation of the unknown control points in case of more data than unknown parameters, cf. Piegl (1991), Sarkar and Menq (1991). In addition, the unknown location parameters of the measured points can be estimated, which leads to a non-linear least squares fit, cf. Lai and Lu (1996). Finally, the weights of the control points may also be considered as unknown parameters of a non-linear adjustment as suggested by Ma and Kruth (1998).

The quality of a fitted NURBS surface depends on the resolution of the measured data. Laserscanners and similar instruments provide a high resolution of the data within a short time, but lead to a huge number of unknown control points. Computationally efficient estimation procedures should therefore be used. The estimation, which is applied here, is a modification of the skinning also called lofting process, cf. Tiller (1983), Piegl (1991). Instead of interpolating a series of cross-sectional curves for obtaining a surface, the curves are being fitted. If there are $n+1$ points of a grid of control points in the direction of the x coordinates and $l+1$ points in the direction of the y coordinates, there are $(n+1) \times (l+1)$ unknown three-dimensional coordinates of the control points to be simultaneously estimated. In case of cross-sectional curve fits two adjustments are needed, the first one with only $n+1$ unknown control points and the second one with only $l+1$ unknown points. Thus, estimates for a large amount of data will be much easier handled, especially if one considers that matrices are sparse in connection with splines. The cross-sectional curve fits of the skinning process are considered an approximation of the simultaneous estimation of the control points (Piegl and Tiller, 1997, p. 419).

However, it was shown by Koch (2009), that both methods give identical results. The simultaneous estimation of the control points should therefore be avoided.
As an example, a surface is fitted to the data of a laserscanner. This problem is often solved by slicing the point cloud measured by the laserscanner to extract sectional contour lines, cf. Yuwen et al. (2006). However, if a laserscanner, for instance Leica HDS 3000, determines the three-dimensional coordinates of points in a grid, cross-sectiomal contour lines follow directly from the lines of the grid. This fact is used here.
In the following Section a short introduction to nonrationl and rational B-spline curves will be given followed by a presentation of B-spline surfaces in Section 3. Section 4 contains simple examples of surfaces fitted by the skinning process to generated data and to measurements of the laserscanner Leica HDS 3000. To judge the accuracy of the estimated surfaces, mean squared differences are computed. The paper finishes with conclusions.

2 B-spline curves

A p th-degree B-spline (basis spline) curve is defined by, cf. Piegl and Tiller (1997, p. 81),

$$\boldsymbol{x}(u) = \sum_{i=o}^{n} N_{i,p}(u)\boldsymbol{p}_i. \tag{1}$$

The point $\boldsymbol{x}(u)$ of the curve is given in a parametric form depending on the parameter u by

$$\boldsymbol{x}(u) = \begin{vmatrix} x(u) \\ y(u) \end{vmatrix} \tag{2}$$

with $x(u)$ and $y(u)$ being the coordinates of $\boldsymbol{x}(u)$ in a plane. The points $\boldsymbol{p}_i = |x_i, y_i|'$ are the control points. They form the control polygon. The B-spline curve follows approximately the shape of this polygon. $N_{i,p}(u)$ with $i \in \{0, \ldots, n\}$ are the p th-degree B-spline basis functions. They are efficiently computed by a recursion formula due to Cox (1972) and de Boor (1972)

$$N_{i,0}(u) = \begin{cases} 1 \text{ if } u_i \le u < u_{i+1} \\ 0 \text{ otherwise} \end{cases}$$
$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{3}$$

where

$$\boldsymbol{u} = |u_0, \ldots, u_m|' \text{ with } u_i \le u_{i+1},\ i \in \{0, \ldots, m-1\} \tag{4}$$

is the $(m+1) \times 1$ knot vector which contains a sequence of nondecreasing real numbers, the knots. The basis functions $N_{i,p}(u)$ represent piecewise polynomials for the half-open interval $u_i \le u < u_{i+1}$ so that the B-spline curve (1) consists of polynomial segments. Evaluating the basis functions by (3) leads to a triangular pattern of dependence. We obtain, for instance, with $p = 3$ by omitting the parameter u

$$\begin{array}{cccc} N_{i,0} & N_{i,1} & N_{i,2} & N_{i,3} \\ & N_{i-1,1} & N_{i-1,2} & N_{i-1,3} \\ & & N_{i-2,2} & N_{i-2,3} \\ & & & N_{i-3,3} \end{array} \tag{5}$$

This pattern indicates that for any knot interval $u_i \le u < u_{i+1}$ there are at most $p+1$ nonzero basis functions, i.e. $N_{i-p,p}, \ldots, N_{i,p}$. For checking the computations of the basis functions the relation

$$\sum_{j=i-p}^{i} N_{j,p}(u) = 1 \text{ for all } u \in [u_i, u_{i+1}) \tag{6}$$

is helpful.
The inverse of the scheme (5) is obtained with (3) for $p = 3$ by the triangular pattern

$$\begin{array}{cccc} N_{i,3} & N_{i,2} & N_{i,1} & N_{i,0} \\ & N_{i+1,2} & N_{i+1,1} & N_{i+1,0} \\ & & N_{i+2,1} & N_{i+2,0} \\ & & & N_{i+3,0} \end{array} \tag{7}$$

It shows that $N_{i,p}(u) = 0$, if u is outside the half-open interval $[u_i,\ u_{i+p+1})$. This indicates the property of local control. If the control point $\boldsymbol{p}_i$ is moved, the curve $\boldsymbol{x}(u)$ changes only for $u \in [u_i,\ u_{i+p+1})$. B-spline curves are therefore well suited for interactive modifications in computer-aided design.
In the following we will work with B-spline curves having the property of endpoint interpolation, which means, $\boldsymbol{x}(u)$ starts at the first control point $\boldsymbol{p}_0$ and ends at the last control point $\boldsymbol{p}_n$. It is obtained with open or nonperiodic knot vectors in contrary to periodic ones. An open knot vector $\boldsymbol{u}$ is given by

$$\boldsymbol{u} = |a, \ldots, a, u_{p+1}, \ldots, u_{m-p-1}, b, \ldots, b|' \quad \text{for } a \le u \le b \tag{8}$$

or

$$\boldsymbol{u} = |0, \ldots, 0, u_{p+1}, \ldots, u_{m-p-1}, 1, \ldots 1|' \quad \text{for } 0 \le u \le 1 \tag{9}$$

where the first and the last knot have multiplicity $k = p + 1$, cf. Piegl and Tiller (1997, p. 81). One could also introduce multiplicity $k = p$, cf. Farin and Hansford (2000, p. 141) and Rogers (2001, p. 51).
When computing a B-spline curve only parameter values within the range of knots

$$u_p, u_{p+1}, \ldots, u_{m-p} \tag{10}$$

are considered. Generally, these interior knots are not equally spaced which leads to the nonuniform B-splines in contrary to the uniform ones which are equally spaced. Interior knots may also have multiplicity up to order p. The number s of piecewise polynomials equals the number of intervals of nonzero lengths within the interior knots. If all interior knots have multiplicity one, it is

$$s = m - 2p. \tag{11}$$

Looking at (5) as well as (8) or (9) it becomes obvious that the number $n+1$ of basis functions, which is equal to the number $n+1$ of control points, follows from the number $m+1$ of knots and the degree p by

$$n = m - p - 1 \tag{12}$$

and therefore with (11)

$$n = s + p - 1. \tag{13}$$

The basis function $N_{i,p}(u)$ is because of (3) a linear combination of two basis functions of degree $p-1$. The B-spline curve $\boldsymbol{x}(u)$ is therefore infinitely differentiable within the knot intervals and at least $p-k$ times continuously differentiable at a knot of multiplicity k so that the curve is $\boldsymbol{x}(u)^{p-1}$ continuous for all interior knots of multiplicity one.
A curve based on p th-degree nonuniform rational B-splines (NURBS) is defined by, c.f. Piegl and Tiller (1997, p. 117).

$$\boldsymbol{x}(u) = \sum_{i=0}^{n} N_{i,p}(u) w_i \boldsymbol{p}_i / \sum_{i=0}^{n} N_{i,p}(u) w_i \tag{14}$$

where w_i denotes the weight associated with the control point $\boldsymbol{p}_i$. As a special case the nonrational B-spline curve (1) is obtained with $w_i = 1$ because of (6). If w_i for $\boldsymbol{p}_i$ is increased, the point $\boldsymbol{x}(u)$ of the curve moves closer to $\boldsymbol{p}_i$ and further away, if the weight is decreased. Due to the local control mentioned in connection with (7), the curve $\boldsymbol{x}(u)$ changes only for $u \in [u_i, u_{i+p+1})$. This property in addition to the local control gives more flexibility for interactive modifications of curves in computer-aided design.
NURBS curves can be efficiently expressed by nonrational B-spline curves, if homogeneous coordinates are used, that is by representing two-dimensional points by points in three dimensions. We introduce the three-dimensional weighted control points

$$\boldsymbol{p}_i^w = |w_i x_i,\ w_i y_i,\ w_i|' = |X_i,\ Y_i,\ W_i|' \tag{15}$$

and accordingly the three-dimensional coordinates $\boldsymbol{x}^w(u)$. The nonrational B-spline curve in three-dimensional space is defined by

$$\boldsymbol{x}^w(u) = \sum_{i=0}^{n} N_{i,p}(u) \boldsymbol{p}_i^w. \tag{16}$$

We now apply perspective mapping, i.e. we map $\boldsymbol{x}^w(u)$ onto the hyperplane $W = 1$. Thus, we divide the first two coordinates of $\boldsymbol{x}^w(u)$ by the third coordinate $W \neq 0$ and obtain the two coordinates

$$\boldsymbol{x}(u) = \left| \frac{X(u)}{W(u)},\ \frac{Y(u)}{W(u)} \right|'. \tag{17}$$

This gives the NURBS curve (14). One can therefore continue to work with the nonrational B-spline curve (1). If a NURBS curve is needed, one introduces the three-dimensional representation (16) and obtains by dividing by the weight w_i the NURBS curve (14).
A point $\boldsymbol{x}(u)$ of the B-spline curve (1) is connected to the control points $\boldsymbol{p}_i$ by a linear relation. If points $\boldsymbol{x}(\bar{u}_o)$ with location parameters $\bar{u}_o$ for $o \in \{1, \ldots, r\}$ are given and we want to approximate them by a p th-degree B-spline curve, the $n+1$ control points $\boldsymbol{p}_i$ for $i \in \{0, \ldots, n\}$ with $r > n+1$ need to be determined. Because of the linear relation, Eq. (1) leads immediately to the observation equations for estimating the unknown control points

$$\begin{aligned} N_{0,p}(\bar{u}_1)\boldsymbol{p}_0 + \ldots + N_{n,p}(\bar{u}_1)\boldsymbol{p}_n &= \boldsymbol{x}(\bar{u}_1) + \boldsymbol{e}_1 \\ N_{0,p}(\bar{u}_2)\boldsymbol{p}_0 + \ldots + N_{n,p}(\bar{u}_2)\boldsymbol{p}_n &= \boldsymbol{x}(\bar{u}_2) + \boldsymbol{e}_2 \\ &\cdots \\ N_{0,p}(\bar{u}_r)\boldsymbol{p}_0 + \ldots + N_{n,p}(\bar{u}_r)\boldsymbol{p}_n &= \boldsymbol{x}(\bar{u}_r) + \boldsymbol{e}_r \end{aligned} \tag{18}$$

where $\boldsymbol{e}_o$ for $o \in \{1, \ldots, r\}$ denotes the vector of errors. Using matrix notation we get

$$\boldsymbol{X}\beta = \boldsymbol{y} + \boldsymbol{e} \tag{19}$$

with $\boldsymbol{X}$ being the $r \times (n+1)$ matrix of B-spline basis functions, $\beta = |\boldsymbol{p}_0, \ldots, \boldsymbol{p}_n|'$ the $(n+1) \times 1$ vector of unknown control points, $\boldsymbol{y} = |\boldsymbol{x}(\bar{u}_1), \ldots, \boldsymbol{x}(\bar{u}_r)|'$ the $r \times 1$ vector of observations and $\boldsymbol{e} = |\boldsymbol{e}_1, \ldots, \boldsymbol{e}_r|'$ the vector of errors. The unknown parameters β are estimated by $\hat{\beta}$ with the normal equations, cf. Koch (1999, p. 158),

$$\boldsymbol{X}'\boldsymbol{X}\hat{\beta} = \boldsymbol{X}'\boldsymbol{y}. \tag{20}$$

The question of determining the parameters $\bar{u}_o$ for the given points $\boldsymbol{x}(\bar{u}_o)$ will be discussed in the following Section 3. After having estimated the control points $\boldsymbol{p}_i$ any point of the curve may be computed by (1). If the curve needs to be changed without changing the control points, a NURBS curve (14) using (16) can be computed with introducing weights for the control points.

3 B-spline surfaces

The B-spline curve according to (1) and (2) is given in a parametric form depending on one parameter. The B-spline surface $\boldsymbol{s}(u, v)$ in three-dimensional space is also introduced in a parametric form depending on the two parameters u and v

$$\boldsymbol{s}(u,v) = \begin{vmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{vmatrix}. \tag{21}$$

Different formulations exist for parametric surfaces, cf. Rogers (2001, p. 6). A representation which is often applied in computer-aided design is the tensor product scheme, cf. Piegl and Tiller (1997, p. 34), which is a bilinear form, cf. Koch (1999, p. 44),

$$\boldsymbol{s}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{l} N_{i,p}(u) N_{j,p}(v) \boldsymbol{p}_{i,j} \tag{22}$$

where $\boldsymbol{s}(u, v)$ is a point on the B-spline surface, $N_{i,p}(u)$ and $N_{j,q}(v)$ are B-spline basis functions of degree p and q and $\boldsymbol{p}_{i,j}$ establish a grid of three-dimensional control points. With $\boldsymbol{s}(u, v = \text{const})$ an isoparametric curve in the direction u on the surface is defined, which shall point along the x axis of the coordinate system, and $\boldsymbol{s}(u = \text{const}, v)$ is an isoparametric curve in the direction v pointing along the y axis.
The B-spline surface (22) consists of patches of bivariate polynomials formed by the rectangles $u_i \leq u \leq u_{i+1}$ and $v_j \leq v \leq v_{j+1}$. The surface follows approximately the grid of control points. If the $(m+1) \times 1$ knot vector $\boldsymbol{u}$ for the parameter u is given by (8) or (9) and the vector $\boldsymbol{v}$ for the parameter v accordingly, the surface interpolates the control points $\boldsymbol{p}_{0,0}, \boldsymbol{p}_{n,0}, \boldsymbol{p}_{0,l}, \boldsymbol{p}_{n,l}$ at the four corners of the grid.

The surface $\boldsymbol{s}(u, v)$ possesses like the B-spline curve the property of local control. If a control point $\boldsymbol{p}_{i,j}$ is moved, only the patches of the surface determined by the rectangle of knot intervals $[u_i, u_{i+p+1})$ and $[v_j, v_{j+q+1})$ are changed. The number s of piecewise polynomials in the direction u follows from (11) and the number in the direction v accordingly. Within the rectangle formed by the knot intervals $[u_i, u_{i+1})$ and $[v_j, v_{j+1})$ the surface $\boldsymbol{s}(u, v)$ is indefinitely differentiable. At the inner knots of the vectors $\boldsymbol{u}$ or $\boldsymbol{v}$ it is $p-k$ or $q-k$ times differentiable in the direction of u or v with k being the multiplicity of the knot.

A surface based on nonuniform rational B-splines (NURBS) is given with (14) and (22) by

$$\boldsymbol{s}(u, v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{l} N_{i,p}(u)N_{j,p}(v)w_{i,j}\boldsymbol{p}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{l} N_{i,p}(u)N_{j,p}(v)w_{i,j}}. \tag{23}$$

As a special case the nonrational B-spline surface (22) follows with $w_{i,j} = 1$ because of (6). A NURBS surface offers in addition to the local control of (22) the possibility to introduce weights $w_{i,j}$ for the control points $\boldsymbol{p}_{i,j}$ by which the shape of the surface can also be locally changed. This is helpful for the interactive construction of surfaces by computer-aided design.

As NURBS curves with (16), NURBS surfaces can be represented by the nonrational B-spline surface (22) in four-dimensional space, if homogeneous coordinates are introduced

$$\boldsymbol{s}^w(u, v) = \sum_{i=0}^{n}\sum_{j=0}^{l} N_{i,p}(u)N_{j,p}(v)\boldsymbol{p}_{i,j}^w \tag{24}$$

with $\boldsymbol{p}_{i,j}^w = |w_{i,j}x_{i,j},\ w_{i,j}y_{i,j},\ w_{i,j}z_{i,j},\ w_{i,j}|'$ being the weighted control points in four-dimensional space. By dividing the first three coordinates by the fourth one for $w_{i,j} \neq 0$ the NURBS surface (23) follows.

Like fitting B-spline curves to given points by (20), surfaces shall now be fitted to measured data. Free-form surfaces will be considered which cannot be represented by simple surfaces like planes, spheres, cones or cylinders. The quality of estimating a surface depends on the resolution of the data by which a surface is determined. If surfaces are fitted to data of laserscanners or similar instruments, a dense net of points can be rapidly measured. Thus, not the resolution is a problem but handling the large amount of data, as already mentioned in the introduction. Cross-sectional curve fits are therefore computed without solving for the location parameters u and v of the measured points and for the weights $w_{i,j}$ of the control points, which leads to nonlinear estimation problems.

Let a grid of $r \times e$ points $\boldsymbol{s}(\bar{u}_o, \bar{v}_d)$ with location parameters $\bar{u}_o$ and $\bar{v}_d$, $o \in \{1, \ldots, r\}$, $d \in \{1, \ldots, e\}$ be measured to estimate the control points $\boldsymbol{p}_{i,j}$ with $i \in \{0, \ldots, n\}$ and $j \in \{0, \ldots, l\}$ of a $(n+1) \times (l+1)$ grid for $r > n+1$ and $e > l+1$. We rewrite (22) to obtain

$$\boldsymbol{s}(\bar{u}_o, \bar{v}_d) = \sum_{i=0}^{n} N_{i,p}(\bar{u}_o)\boldsymbol{f}_{i,d} \quad \text{for } d \in \{1, \ldots, e\} \tag{25}$$

with

$$\boldsymbol{f}_{i,d} = \sum_{j=0}^{l} N_{j,q}(\bar{v}_d)\boldsymbol{p}_{i,j} \tag{26}$$

where $\boldsymbol{f}_{i,d}$ are the control points of the isoparametric curve $\boldsymbol{s}(u, v = \text{const})$ on the surface. The control points $\boldsymbol{p}_{i,j}$ are estimated in two steps. The first one consists of estimating $\boldsymbol{f}_{i,d}$ by using (25) as observation equations like (18) for fitting e times a B-spline curve to the given r data points $\boldsymbol{s}(\bar{u}_o, \bar{v}_d)$ for $o \in \{1, \ldots, r\}$ with $d \in \{1, \ldots, e\}$. Only one Cholesky factorization of the normal equations is needed, the e back solutions give the three coordinates of the points $\boldsymbol{f}_{i,d}$ for $d \in \{1, \ldots, e\}$, cf. Koch (1999, p. 30). The approximately constant y cordinates for each of the e adjustments enter as observations and follow also as estimates because of (6).

The observation equations for the second step of the parameter estimation are obtained from (26). They serve for fitting B-spline curves $(n+1)$ times to the e given points $\boldsymbol{f}_{i,d}$ for $d \in \{1, \ldots, e\}$ so that the control points $\boldsymbol{p}_{i,j}$ with $i \in \{0, \ldots, n\}$ and $j \in \{0, \ldots, l\}$ are estimated. Thus, the skinning process consists of cross-sectional curve fits along isoparametric lines first in the u and then in the v direction, cf. Tiller (1983), Piegl (1991). Instead of solving for $(n+1) \times (l+1)$ unknown control points, two estimates are computed the first one with $n+1$ unknown parameters, the second one with $l+1$ unknown parameters. This gives a simple method of estimating B-spline surfaces, whose results agree with the simultaneous estimates of the control points as mentioned in the introduction. It preserves the accuracy of the data as will be shown in the next Section 4. Any point on the B-spline surface can be computed by (25) and (26) from the estimated control points $\boldsymbol{p}_{i,j}$. If this surface needs to be modified, weights $w_{i,j}$ are introduced for the control points so that a NURBS surface is computed by (24). This will be demonstrated in Section 4.

The x, y and z coordinates of the grid of points $\boldsymbol{s}(\bar{u}_o, \bar{v}_d)$ are measured. As mentioned above, the isoparametric curve in the direction u points along the x axis and the isoparametric curve in the direction v along the y axis. The unknown parameters $\bar{u}_o$ are determined by the chord lengths, i.e. by the distances of the points on the isoparametric curve in the direction of the x axis for each value y of the grid. The mean over all values for y gives $\bar{u}_o$ for $o \in \{1, \ldots, r\}$. Correspondingly, the parameters $\bar{v}_d$ are determined. The inner knots u_i and v_j in the u and v directions are evenly distributed. The fitted surface should capture the shape of the data. It may not oscillate between the data points but smooth the variances of the data. This is accomplished by choosing the number of polynomial segments in the u and v direction smaller than the number r and e of data points in the x and y direction.

4 Numerical examples

To test the method (25) and (26) for fitting surfaces to data, the z coordinates of points for a grid of x and y coordinates have been generated representing a surface within a rectangle of $x = 7$ m and $y = 5$ m. The z coordinates are computed by the probability density function of a bivariate normal distribution multiplied by 100 with the maximum height of 3.03 m at $x = 4$ m and $y = 3$ m. It is assumed that the z coordinates represent measurements with standard deviations of 1 cm. Random variates from the normal distribution with expected values 0 cm and standard deviations 1 cm have therefore been added to the z coordinates, cf. KOCH (2007, p. 197).

For the first example the measured z coordinates for a grid of 15×11 points are given and shown in Fig. 1. A surface is fitted to the data by (25) and (26) with $p = q = 3$ and 7 polynomial segments in x direction and 5 segments in y direction. Thus, the unknown control points form a grid of 10×8 points because of (13). The standard deviation of the z coordinates resulting from the parameter estimation is 0.96 cm which is approximately equal to the standard deviation of 1 cm of the measured z coordinates, see first line of Table 1.

To check whether the adjusted surface does not oscillate between the data points but catches its shape, the coordinates of a grid of 36×26 points are computed with the estimated control points by (25) and (26) and shown in Fig. 2. These points are chosen such that they do not coincide with the data points. The differences for the 36×26 points between the z coordinates of the adjusted surface and the z coordinates times 100 of the bivariate normal distribution for these points lie between -2.17 cm and 2.09 cm. The square root of the mean squared differences is 0.89 cm, see first line of Table 1. These results together with Fig. 2 show that the fitted surface represents the surface generated by the bivariate normal distribution very well. By introducing more data points by a grid of 22×16 and of 29×21 points the surface fits can be slightly improved as shown by the results of the second and third line of Table 1.

As a further example, a standing tube for ventilation made of concrete was chosen with the outside having the shape of an octagon, whose planes meet in perpendicular rough edges. The coordinates of the octagon have been measured by the laserscanner Leica HDS 3000. Three planes are visible from the position chosen for the laserscanner, and they are scanned by 47 points in the horizontal x direction and 11 points in the vertical z direction, see Fig. 3. The height is now representd by the y coordinate and not by the z coordinate as before. The two rough edges were not scanned. The distances from the instrument to the octagon vary from 6.22 m to 7.28 m.

As can be seen from Fig. 3 the scanned points do not lie exactly on straight lines due to deviations of the surface of the octagon from planes. A free-form surface is fitted by

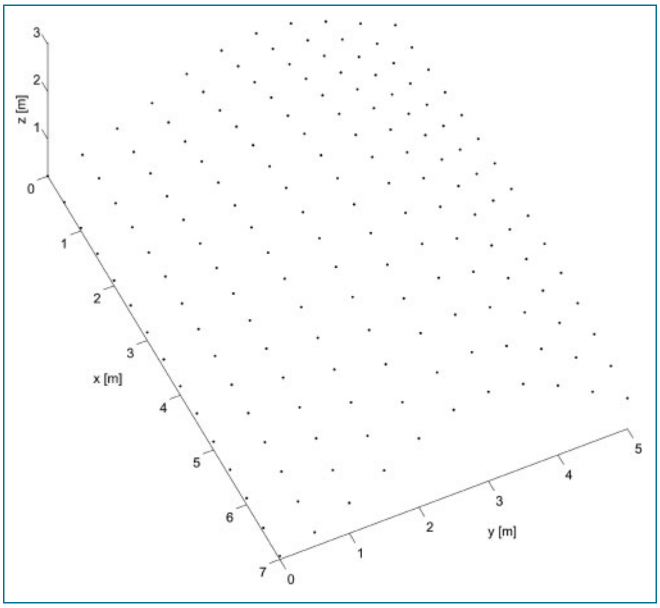

Fig. 1: Grid of 15 × 11 points with z coordinates having standard deviations of 1 cm

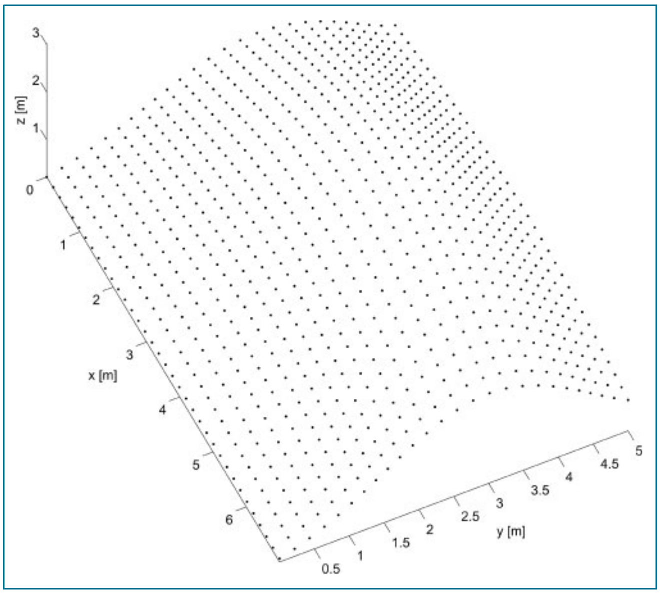

Fig. 2: Grid of 36 × 26 points on the fitted surface

(25) and (26) to the scanned data with $p = q = 3$ and 14 polynomial segments in x direction and 4 in z direction so that a grid of 17×7 unknown control points has to be estimated. The standard deviation of this adjustment results in 0.5 cm, which represents the accuracy of the measured

Table 1: Results for fitting surfaces to three different grids of points

Grid of points		Polynomial segments		Std. dev. measurem.	Points on fitted surface		Dev. in z min.	max.	Sqrt. of mean sq. dev. in z
x	y	x	y	[cm]	x	y	[cm]	[cm]	[cm]
15	11	7	5	0.96	36	26	– 2.17	2.09	0.89
22	16	7	5	0.99	36	26	– 2.08	1.95	0.77
29	21	9	7	1.02	36	26	– 1.89	1.85	0.62

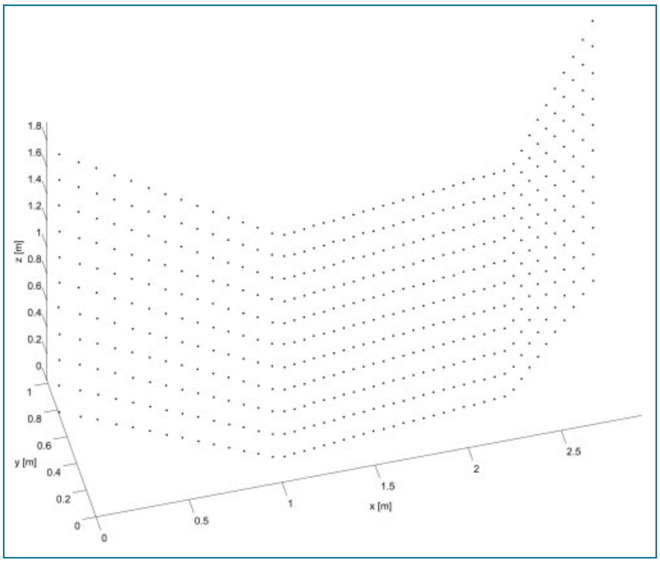

Fig. 3: Grid of 47 × 11 points measured by the laserscanner Leica HDS 3000

y coordinates, because for distances of about 4.70 m to a well reflecting plane surface a standard deviation of the y coordinates of about 0.3 cm was obtained for the same instrument (Koch, 2008a). The remaining 0.2 cm can be attributed to the rough surface of concrete. The coordinates of a grid of 74 × 17 points on the fitted surface, which do not coincide with the data points, are computed by the estimated control points.

To check whether this surface catches the shape of the measured data, a plane is fitted to the scanned points of the middle plane of the octagon. The x and z coordinates of this adjustment are held fixed, because the variances of these coordinates are much smaller than the ones of y coordinates (Koch, 2008a). The standard deviation of the y coordinates from fitting the plane is 0.6 cm. Is is slightly larger than the standard deviation of 0.5 cm for fitting the surface due to deviations, mentioned above, of the surface of the octagon from the surface of the plane. The differences between the y coordinates of the surface fit and the ones of the adjusted plane vary between −4.5 cm and 1.4 cm and the square root of the mean squared differences equals 1.0 cm.

The fitted surface can be moved closer to the adjusted plane by computing the NURBS surface (23). The weights for the estimated control points are set to $w_{i,j} = 1$ except for the two lines of control points with varying z values and same x values at both ends of the middle plane which get $w_{i,j} = 1.6$. The NURBS surface is computed for the same grid of 74 × 17 points mentioned above and shown in Fig. 4. As can be seen the two edges where the three planes meet are rounded and the irregularities of the planes shown in Fig. 3 are smoothed. The differences of the y coordinates of the NURBS surface and the ones of the fitted plane now are smaller and vary between −2.4 cm and 2.2 cm. The square root of the mean squared differences is 0.8 cm. Its discrepancy from the standard deviation of 0.5 cm of the surface fit is caused by the fact that the scanned data does not lie exactly in a plane because of the rough concrete. The results therefore show that fitting a NURBS surface to scanned data can be achieved with an accuracy which approximates the accuracy of the laserscanner.

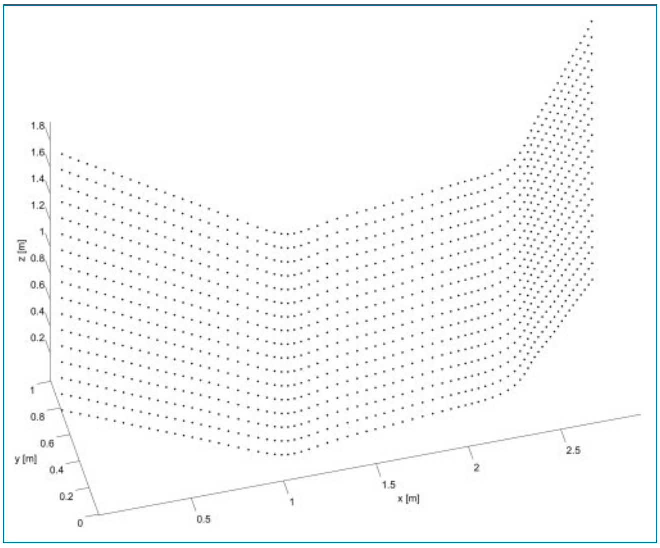

Fig. 4: Grid of 74 × 17 points on the fitted NURBS surface

5 Conclusions

It is shown that the skinning process leads to an efficient method for fitting free-form surfaces to measured data. Numerical results for simple examples of generated data and measurements of the laserscanner Leica HDS 3000 indicate that the surfaces are fitted with an accuracy that approximates the one of the data. This has been shown by computing standard deviations and mean squared differences. For more elaborate error studies Monte Carlo simulations are needed as applied to independent measurements by Koch (2008b) and to correlated data by Koch (2008a).

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Abstract

The task of fitting free-form surfaces to data of laserscanners or similar instruments has to be solved not only in reverse engineering but also in engineering geodesy. It leads to almost as many unknown parameters as data points. A fast method for fitting surfaces by NURBS (nonuniform rational B-splines) is applied by estimating cross-sectional curves. It needs approximately only the square root of the number of data points as unknown parameters. It is shown for simple examples of generated data and data of the laserscanner Leica HDS 3000 that the free-form surfaces can be fitted with an accuracy which approximates the accuracy of the measured data.

Zusammenfassung

Nicht nur im Maschinenbau, auch in der Ingenieurgeodäsie muss die Aufgabe gelöst werden, Freiformflächen an Daten von Laserscannern oder ähnlichen Instrumenten anzupassen. Dabei treten fast ebenso viele unbekannte Parameter wie Datenpunkte auf. Ein schnelles Verfahren zur Anpassung von Flächen mit NURBS (nonuniform rational B-splines) wird daher mit Hilfe der Schätzung sich überkreuzender Kurven angewendet. Es benötigt näherungsweise nur die Wurzel aus der Anzahl der Datenpunkte als unbekannte Parameter. Mit einfachen Beispielen generierter Messungen und Messungen des Laserscanners Leica HDS 3000 wird gezeigt, dass die Anpassung der Freiformflächen mit einer Genaugkeit erfolgen kann, die die Genauigkeit der gemessenen Daten approximiert.