NURBS Surface with Changing Shape

Karl Rudolf Koch

ALLGEMEINE VERMESSUNGS-NACHRICHTEN

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Die Anpassung von Freiformflächen an die gemessenen Koordinaten von Punkten an der Oberfläche von Objekten wird in allgemeinen durch das Tensorprodukt von NURBS (nonuniform rational B-splines) gelöst. NURBS Flächen werden in ‚reverse engineering' und in der Ingenieurgeodäsie benötigt. Die NURBS Fläche wird hier derart verallgemeinert, dass eine Fläche mit sich ändernder Gestalt erhalten wird. Sie lässt sich für die Deformationsanalyse anwenden, wenn die Gestalt der Fläche zu Zeitpunkten benötigt wird, an denen keine Messungen vorliegen. Man kann auch Flächen darstellen, die von Punkten mit dreidimensionalen Koordinaten abhängen. Die Koordinaten von Punkten an der Oberfläche einer dünnen Platte aus Plastik werden mit dem Laserscanner Leica HDS 3000 gemessen. Die Platte wird unter wachsendem Druck gebogen, und die Koordinaten für verschiedene Werte des Drucks gemessen. Die verallgemeinerte NURBS Fläche wird durch Schätzung sich überkreuzender Kurven an die Messungen angepasst. Die Punkte auf der deformierten Platte werden für Werte des Drucks berechnet, die zwischen den Werten des Drucks für die Messungen liegen.

1 Introduction

Three-dimensional coordinates of points on the surface of an object are measured with high accuracy, speed and resolution by laserscanners or similar instruments. A free-form surface is fitted to the measured points to obtain an analytical model of the object. This is an essential step in reverse engineering where an analytical model of a manufactured object is used for computer-aided design, cf. Varady et al. (1997), Ma and Kruth (1998). Analytical models of objects are also applied in engineering geodesy, for instance, to correct for the offsets of the reflectors of lasertrackers (Hennes, 2009) or for examining surfaces of roads (Koch, 2009c). Free-form surfaces are generally represented by the tensor product of two nonuniform rational B-splines (NURBS), cf. Piegl and Tiller (1997, p. 34), Koch (2009a). Such a representation depends on two parameters so that a two-dimensional NURBS surface is defined. The points on the NURBS surface are expressed by three-dimensional coordinates.

A surface based on the tensor product of nonuniform nonrational B-splines includes the representation of a NURBS surface if homogeneous coordinates are introduced, cf. Koch (2009a). In the following, we will therefore only work with nonrational B-spline surfaces.

Engineering geodesy frequently deals with surfaces of objects which change with time, for instance, in deformation analysis. Coordinates of points on the surface are measured at different time epochs to register the deformation. A free-form surface can then be fitted to the data to obtain the deformation at the different time epochs. If the shape of the surface is needed between the time epochs of the measurements, interpolation has to be applied. In general, a linear interpolation will not be sufficient so that some computational effort has to be invested.

A B-spline surface defined by the tensor product is easily generalized from a two-dimensional surface depending on two parameters to a three-dimensional surface as a function of three parameters by adding an additional summation and a B-spline basis function. The coordinates of points on this surface are expressed in four dimensions with the third coordinate being, for instance, the time and the fourth coordinate the height, if the B-spline surface changes with time. In case a quantity like the temperature is measured in three-dimensional space, the three-dimensional B-spline surface can be used, if three coordinates express the position in space and the fourth coordinate contains the temperature. When the temperature changes with time and is measured at different time epochs, a four-dimensional B-spline surface may be defined with five-dimensional coordinates: the three positions, the time and the temperature.

The representation of a B-spline surface by the tensor product gives a linear relation between the measured coordinates of points on the surface of an object and the unknown control points of the surface, if the knots of the B-spline basis functions are selected and the local parameters of the measured points are determined. The linear relation leads to the observation equations of a linear model where the control points are simultaneously estimated by least-squares adjustment. This holds for a two-dimensional B-spline surface, cf. PIEGL (1991), FARIN and HANSFORD (2000, p. 186), ROGERS (2001, p. 193), but also for a three-dimensional surface as shown by SCHMIDT (2007) and ZEILHOFER et al. (2009). They estimated simultaneously the control points of a B-spline surface for the electron density of the ionosphere, which is a function of three-dimensional position and of time. Surfaces at different time epochs were determined.

When increasing the dimensions of a B-spline surface, the number of unknown control points increases. A two-dimensional B-spline surface is generally determined by a grid of control points. They are efficiently estimated by a Cholesky factorization. If $k \times k$ control points are simultaneously estimated, the Cholesky factorization of the $k^2 \times k^2$ matrix of normal equations requires a computational complexity of $O(k^6)$ (KOCH, 2009b). When applying, e.g. for a deformation analysis, a three-dimensional B-spline surface and when $k \times k \times k$ control points have to be simultaneously estimated, the numerical complexity increases to $O(k^9)$ (KOCH, 2010). This might, depending on k, result in a heavy computational burden.

Estimating the unknown control points by cross-sectional curve fits is much faster than the simultaneous estimation. This procedure is called lofting or skinning method (TILLER, 1983), (PIEGL, 1991), and also approximate lofting (PARK, 2001). Its computational complexity is only $O(k^3)$ for two-dimensional B-spline surfaces (KOCH, 2009b). However, the lofting method is considered to be an approximation of the simultaneous estimation of the control points PIEGL and TILLER (1997, p. 419). But it was proved by KOCH (2009b) that the lofting method and the simultaneous estimation give identical results. Estimating k^3 control points of a three-dimensional B-spline surface by the lofting method requires a numerical complexity of only $O(k^4)$ (KOCH, 2010), which computationally is still manageable. This is important, if the accuracy of fitting B-spline surfaces is investigated by Monte Carlo simulations (KOCH, 2009c). Furthermore, it was shown by (KOCH, 2010) that the estimates of the control points by the lofting method and the simultaneous estimation give identical results for three-dimensional B-spline surfaces also.

The three-dimensional B-spline surface is applied here to determine the deformation of a thin sheet bent under pressure. In the following Section the three-dimensional B-spline surface is therefore defined, and Section 3 presents the lofting method by cross-sectional curve fits for estimating the unknown control points. The coordinates of a grid of points on the deformed surface of the sheet are measured by a laserscanner for different values of the pressure. It is important to know especially for engineering geodesy whether the free-form surface is determined with an accuracy which approximates the accuracy of the measurements. Thus, Section 4 deals with obtaining the variances of the coordinates measured by the laserscanner. Section 5 gives the results of the deformation of the sheet. The paper finishes with conclusions.

2 Three-dimensional B-spline surface

The three-dimensional B-spline surface is a function of the three parameters *u, v, w* and is expressed by the tensor product of the three B-spline basis functions $N_{ip}(u),\ N_{jq}(v),\ N_{kr}(w)$ of degree *p, q, r* with

$$\boldsymbol{s}(u,v,w) = \sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n} N_{ip}(u)N_{jq}(v)N_{kr}(w)\boldsymbol{p}_{ijk} \tag{1}$$

and

$$\boldsymbol{s}(u,v,w) = \begin{vmatrix} x(u,v,w) \\ y(u,v,w) \\ z(u,v,w) \\ h(u,v,w) \end{vmatrix} \tag{2}$$

where *x, y, z* denote three-dimensional rectangular or curvilinear coordinates depending on the three parameters *u, v, w* and *h* the height of the B-spline surface. The points $\boldsymbol{p}_{ijk} = |x_i,\ y_j,\ z_k,\ h_{ijk}|$ with $i \in \{0,\ldots,l\}$, $j \in \{0,\ldots,m\},\ k \in \{0,\ldots,n\}$ are the control points. They can be visualized as spatially arranged in grids. The B-spline surface approximately follows the control points.

The B-spline basis functions are efficiently computed by a recursion formula due to COX (1972) and DE BOOR (1972)

$$N_{i0}(u) = \begin{cases} 1 & \text{if } u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{ip}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{3}$$

where

$$\boldsymbol{u} = |u_0,\ldots,u_o|' \quad \text{with} \quad u_i \le u_{i+1}, i \in \{0,\ldots,o-1\} \tag{4}$$

is the $(o+1) \times 1$ vector of the knots u_i with $l = o - p - 1$. Accordingly, $N_{jq}(v)$ and $N_{kr}(w)$ are computed. The basis functions $N_{ip}(u)$ represent piecewise polynomials for the interval $u_i \le u < u_{i+1}$, and the number of polynomial segments is $o - 2p$, cf. KOCH (2009a). The knots in general are not equally spaced giving non-uniform B-splines in contrary to uniform B-splines with equally spaced knots. Open or nonperiodic knot vectors in contrary to periodic ones are introduced with the property of endpoint interpolation. This means that the B-spline surface goes through the corners of the spatial grid of control points $\boldsymbol{p}_{ijk}$.

The points $\boldsymbol{s}(u, v = \text{const}, w = \text{const})$ with *u* variable and *v, w* fixed define an isoparametric curve as a function of *u* on the B-spline surface. Accordingly, $\boldsymbol{s}(u = \text{const}, v, w = \text{const})$ and $\boldsymbol{s}(u = \text{const}, v = \text{const}, w)$ are isoparametric curves depending on *v* and *w*. The isoparametric curve for *u* shall point approximately along the *x* axis, the one for *v* along the *y* axis and for *w* along the *z* axis.

3 Estimation by lofting method

Let the three-dimensional rectangular or curvilinear coordinates of $d \times e \times f$ points $\boldsymbol{s}(u_a,\ v_b,\ w_c)$ be measured where u_a with $a \in \{1,\ldots,d\}$, v_b with $b \in \{1,\ldots,e\}$, w_c with $c \in \{1,\ldots,f\}$ denote the local parameters. The points $\boldsymbol{s}(u_a,\ v_b,\ w_c)$ are spatially arranged in grids of arbitrary shape formed by the measuring process. The three-dimensional B-spline surface (1) shall be fitted to the measured coordinates so that the control points $\boldsymbol{p}_{ijk}$ with $i \in \{0,\ldots,l\}, j \in \{0,\ldots,m\}, k \in \{0,\ldots,n\}$ have to be estimated for $d > l+1,\ e > m+1$ and $f > n+1$. The local parameters $u_a,\ v_b,\ w_c$ are generally determined by the chord lengths of the measured points (Piegl and Tiller, 1997, p. 364). To determine u_a with $a \in \{1,\ldots,d\}$, the distances s between the points in the direction of the x axis of the spatial grid are computed for fixed values of y and z

$$s = [(x(u,v,w) - x(u-1,v,w))^2 + (h(u,v,w) - h(u-1,v,w))^2]^{1/2}. \tag{5}$$

This is repeated for all values of y of the grid and the mean of the distances is formed over all values of y. This procedure is repeated again for all values of z and the mean of these values gives u_a for $a \in \{1,\ldots,d\}$. Accordingly, the local parameters v_b and w_c are computed. After choosing the knot vectors for the knots $u_i,\ v_j,\ w_k$, (1) gives a linear relation between the unknown control points $\boldsymbol{p}_{ijk}$ and the measured points $\boldsymbol{s}(u_a,\ v_b,\ w_c)$ so that the observation equations for estimating $\boldsymbol{p}_{ijk}$ in a linear model are obtained

$$\sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n} N_{ip}(u_a)N_{jq}(v_b)N_{kr}(w_c)\boldsymbol{p}_{ijk} = \boldsymbol{s}(u_a,\ v_b,\ w_c) + \boldsymbol{e}(u_a,\ v_b,\ w_c),$$
$$a \in \{1,\ldots,d\}, b \in \{1,\ldots,e\}, c \in \{1,\ldots,f\} \tag{6}$$

where $\boldsymbol{e}(u_a,\ v_b,\ w_c)$ denotes the vector of errors of the measured coordinates. Eq. (6) gives $d \times e \times f$ linear equations for determining $(l+1) \times (m+1) \times (n+1)$ unknown control points.

Instead of simultaneously estimating the control points, the lofting method by cross-sectional curve fits is applied. Eq. (6) is therefore rewritten by

$$\sum_{i=0}^{l} N_{ip}(u_a)\boldsymbol{g}_{ibc} = \boldsymbol{s}(u_a,\ v_b,\ w_c) + \boldsymbol{e}(u_a,\ v_b,\ w_c) \tag{7}$$

with

$$\sum_{j=0}^{m} N_{jq}(v_b)\boldsymbol{h}_{ijc} = \boldsymbol{g}_{ibc} \tag{8}$$

and

$$\sum_{k=0}^{n} N_{kr}(w_c)\boldsymbol{p}_{ijk} = \boldsymbol{h}_{ijc} \tag{9}$$

where $\boldsymbol{g}_{ibc}$ denotes the control points of the isoparametric curves $\boldsymbol{s}(u, v = \text{const}, w = \text{const})$ and $\boldsymbol{h}_{ijc}$ the control points of the isoparametric curves $\boldsymbol{s}(u = \text{const}, v, w = \text{const})$.

In the first step, the control points $\boldsymbol{g}_{ibc}$ are estimated by means of the observation equations (7). They read using matrix notation

$$\boldsymbol{N}(u)\boldsymbol{G} = \boldsymbol{S} + \boldsymbol{E} \tag{10}$$

where the $d \times (l+1)$ matrix $\boldsymbol{N}(u)$ of the B-spline basis functions is defined by

$$\boldsymbol{N}(u) = \begin{vmatrix} N_{0p}(u_1) & \ldots & N_{lp}(u_1) \\ N_{0p}(u_d) & \ldots & N_{lp}(u_d) \end{vmatrix}, \tag{11}$$

the $(l+1) \times (e \times f)$ matrix $\boldsymbol{G}$ of control points by

$$\boldsymbol{G} = \begin{vmatrix} \boldsymbol{g}_{011} & \ldots & \boldsymbol{g}_{0e1} & \ldots & \boldsymbol{g}_{01f} & \ldots & \boldsymbol{g}_{0ef} \\ \boldsymbol{g}_{l11} & \ldots & \boldsymbol{g}_{le1} & \ldots & \boldsymbol{g}_{l1f} & \ldots & \boldsymbol{g}_{lef} \end{vmatrix} = |\boldsymbol{G}_1,\ldots,\boldsymbol{G}_f|, \tag{12}$$

the $d \times (e \times f)$ matrix $\boldsymbol{S}$ of measured points by

$$\boldsymbol{S} = \begin{vmatrix} \boldsymbol{s}(u_1,v_1,w_1) & \ldots & \boldsymbol{s}(u_1,v_e,w_1) & \ldots & \boldsymbol{s}(u_1,v_1,w_f) & \ldots & \boldsymbol{s}(u_1,v_e,w_f) \\ \boldsymbol{s}(u_d,v_1,w_1) & \ldots & \boldsymbol{s}(u_d,v_e,w_1) & \ldots & \boldsymbol{s}(u_d,v_1,w_f) & \ldots & \boldsymbol{s}(u_d,v_e,w_f) \end{vmatrix} \tag{13}$$

and finally the $d \times (e \times f)$ matrix $\boldsymbol{E}$ of errors by

$$\boldsymbol{E} = \begin{vmatrix} \boldsymbol{e}(u_1,v_1,w_1) & \ldots & \boldsymbol{e}(u_1,v_e,w_1) & \ldots & \boldsymbol{e}(u_1,v_1,w_f) & \ldots & \boldsymbol{e}(u_1,v_e,w_f) \\ \boldsymbol{e}(u_d,v_1,w_1) & \ldots & \boldsymbol{e}(u_d,v_e,w_1) & \ldots & \boldsymbol{e}(u_d,v_1,w_f) & \ldots & \boldsymbol{e}(u_d,v_e,w_f) \end{vmatrix}. \tag{14}$$

Eq. (10) represents the observation equations of a multivariate linear model by which the control points $\boldsymbol{g}_{ibc}$ of the isoparametric curves $\boldsymbol{s}(u,\ v = \text{const},\ w = \text{const})$ are estimated e times for each value of v and then f times for each value of w. The estimate $\hat{\boldsymbol{G}}$ of $\boldsymbol{G}$ follows with assuming independent measurements with weights equal to one by, cf. Koch (1999, p. 241),

$$\hat{\boldsymbol{G}} = (\boldsymbol{N}(u)'\boldsymbol{N}(u))^{-1}\boldsymbol{N}(u)'\boldsymbol{S}. \tag{15}$$

The matrix $\boldsymbol{N}(u)$ has full column rank so that the matrix $\boldsymbol{N}(u)'\boldsymbol{N}(u)$ of normal equations is regular if the measured points are evenly distributed like on grids.
Although the estimates $\hat{\boldsymbol{g}}_{ibc}$ of the control points $\boldsymbol{g}_{ibc}$ are correlated, they are substituted as independent measurements in (8) to obtain in the next step the observation equations for estimating the unknown control points $\boldsymbol{h}_{ijc}$ of the isoparametric curves $\boldsymbol{s}(u = \text{const}, v,\ w = \text{const})$

$$\sum_{j=0}^{m} N_{jq}(v_b)\boldsymbol{h}_{ijc} = \hat{\boldsymbol{g}}_{ibc} + \boldsymbol{e}_g. \tag{16}$$

Otherwise, the estimates of the lofting method will not be identical with the simultaneous estimation (Koch, 2010). The estimates $\hat{\boldsymbol{h}}_{ijc}$ of $\boldsymbol{h}_{ijc}$ are obtained corresponding to (15). In the third and final step, the estimates $\hat{\boldsymbol{h}}_{ijc}$ are substituted in (9) to obtain the observation equations for estimating the control points $\boldsymbol{p}_{ijk}$

$$\sum_{k=0}^{n} N_{kr}(w_c)\boldsymbol{p}_{ijk} = \hat{\boldsymbol{h}}_{ijc} + \boldsymbol{e}_h. \tag{17}$$

The estimates $\hat{\boldsymbol{p}}_{ijk}$ of $\boldsymbol{p}_{ijk}$ follow again corresponding to (15).
The residuals, i.e. the estimates $\hat{\boldsymbol{e}}(u_a,\ v_b,\ w_c)$ of the errors $\boldsymbol{e}(u_a,\ v_b,\ w_c)$ in (7) are needed to compute the variance factor of the estimation. It gives the variances of the measurements, cf. Koch (2007, p. 85). By sbstituting $\hat{\boldsymbol{p}}_{ijk}$ in (17) to obtain $\hat{\hat{\boldsymbol{h}}}_{ijc}$

$$\hat{\hat{\boldsymbol{h}}}_{ijc} = \sum_{k=0}^{n} N_{kr}(w_c)\hat{\boldsymbol{p}}_{ijk}, \tag{18}$$

by substituting $\hat{\hat{\boldsymbol{h}}}_{ijc}$ in (16) to find $\hat{\hat{\boldsymbol{g}}}_{ibc}$

$$\hat{\hat{\boldsymbol{g}}}_{ibc} = \sum_{j=0}^{m} N_{jq}(v_b)\hat{\hat{\boldsymbol{h}}}_{ijc}, \tag{19}$$

we obtain the vector $\hat{\boldsymbol{e}}(u_a,\ v_b,\ w_c)$ of residuals from (7) by

$$\hat{\boldsymbol{e}}(u_a,\ v_b,\ w_c) = \sum_{i=0}^{l} N_{ip}(u_a)\hat{\hat{\boldsymbol{g}}}_{ibc} - \boldsymbol{s}(u_a,\ v_b,\ w_c). \tag{20}$$

With

$$\hat{\boldsymbol{e}}(u_a,\ v_b,\ w_c) = \begin{vmatrix} \hat{e}_x(u_a,\ v_b,\ w_c) \\ \hat{e}_y(u_a,\ v_b,\ w_c) \\ \hat{e}_z(u_a,\ v_b,\ w_c) \\ \hat{e}_h(u_a,\ v_b,\ w_c) \end{vmatrix} \tag{21}$$

from (2), the variance factor σ_x^2 for the coordinates x then follows by

$$\sigma_x^2 = \sum_{a=1}^{d}\sum_{b=1}^{e}\sum_{c=1}^{f} \hat{e}_x^2(u_a,\ v_b,\ w_c)/$$
$$(def - (l+1)(m+1)(n+1)) \tag{22}$$

and accordingly $\sigma_y^2,\ \sigma_z^2,\ \sigma_h^2$. These variance factors from fitting the B-spline surface to the measurements give the variances of the measured coordinates because the weights are equal to one as assumed for (15). The variance σ_p^2 for positioning a point is obtained by

$$\sigma_p^2 = \sigma_x^2 + \sigma_y^2 + \sigma_h^2 \tag{23}$$

if the heights h are referred to the x, y plane.
If points $\boldsymbol{s}(u_p,\ v_p,\ w_p)$ with local parameters $u_p,\ v_p,\ w_p$ shall be computed on the estimated B-spline surface, the B-spline basis functions $N_{ip}(u_p),\ N_{jq}(v_p),\ N_{kr}(w_p)$ need to be computed. Eq. (18), (19) and (20) then give

$$\hat{\hat{\boldsymbol{h}}}_{ijcp} = \sum_{k=0}^{n} N_{kr}(w_p)\hat{\boldsymbol{p}}_{ijk}, \tag{24}$$

$$\hat{\hat{\boldsymbol{g}}}_{ibcp} = \sum_{j=0}^{m} N_{jq}(v_p)\hat{\hat{\boldsymbol{h}}}_{ijcp}, \tag{25}$$

$$\boldsymbol{s}(u_p,\ v_p,\ w_p) = \sum_{i=0}^{l} N_{ip}(u_p)\hat{\hat{\boldsymbol{g}}}_{ibcp}. \tag{26}$$

Any number of points on the estimated surface may be computed by these equations.

4 Variances of the measurements

The variances of the measured coordinates are estimated with (22) by fitting the B-spline surface to the measurements. These estimates also reflect modeling errors which occur when the B-spline surface cannot represent the measured coordinates. To find out whether modeling errors have happened, the variances of the measurements are independently estimated.
Variances of measurements can simply be determined by repetitions. The appropriate model to analyze repeated observations is the multivariate model by which variances and covariances of measurements are estimated (Koch, 1999, p. 250), (Koch, 2008). We are interested here in variances only. We assume that the heights h defined by (2) of the B-spline surface are measured with respect to the x, y plane. A grid of points with $z = \text{const}$ is selected and the coordinates $x,\ y,\ h$ of the points of the grid are measured n_w times. Let n_g be the number of points of the grid so that the coordinate $\boldsymbol{s}_{ij}$ are obtained

$$\boldsymbol{s}_{ij} = \begin{vmatrix} x_{ij} \\ y_{ij} \\ h_{ij} \end{vmatrix},\ i \in \{1, \ldots, n_w\},\ j \in \{1, \ldots, n_g\}. \tag{27}$$

The variance σ_{xj}^2 of the grid point j follows with

$$\sigma_{xj}^2 = \sum_{i=1}^{n_w}(x_{ij} - \sum_{i=1}^{n_w} x_{ij}/n_w)^2/(n_w - 1) \tag{28}$$

and accordingly σ^2_{yj} and σ^2_{hj}. It is assumed that the measurements, for instance by a laserscanner, of the coordinates x, y, h of the points of the grid proceed under similar conditions. We therefore take the mean of the variances σ^2_{xj}, σ^2_{yj} and σ^2_{hj} over all points of the grid and obtain the variance σ^2_x of the coordinates x by

$$\sigma^2_x = \sum_{j=1}^{n_g} \sigma^2_{xj}/n_g \tag{29}$$

and accordingly σ^2_y of y and σ^2_h of h. The variance for positioning a point follows from (23). The variances obtained by (29) are compared with the ones by (22) to find out whether the B-spline surface is computed with an accuracy which approximates the accuracy of the measurements and whether modeling errors occur when fitting the B-spline surface.

5 Deformation of a B-spline surface

A thin plastic sheet is deformed under pressure. The heights h in (2) of the sheet are defined with respect to an x, y plane. The coordinates z are therefore identified with the pressure p_s, $z = p_s$. The sheet with a well reflecting surface is of rectangular shape and fixed at one side with $y = -0.30$m, see Fig. 1 and 2, where the coordinates x, y, h are given in meters. It is bent over a sharp edge at the opposite side with $y = 0.12$m. The pressure is increased in seven steps and numbered, $p_s \in \{1, 2, \ldots, 7\}$. For each value of the pressure p_s, the coordinates of a grid of 18×28 points are measured, altogether $18 \times 28 \times 7$ points, by the laserscanner Leica HDS 3000 of the Institute of Geodesy and Geoinformation. Fig. 1 shows the measured points at pressure $p_s = 1$ and Fig. 2 at $p_s = 7$. The rectangular coordinates $\bar{x}$, $\bar{y}$, $\bar{z}$ of the laserscanner have their origin at the center of the instrument. The $\bar{x}$ axis lies horizontally, the $\bar{y}$ axis coincides with the center of lines of sight of the instrument and the $\bar{z}$ axis points to the zenith. All figures show the coordinates defined by (2). They are obtained from the coordinates of the laserscanner by setting $x = \bar{x}$, $y = \bar{z}$ and $h = \bar{y}_m - \bar{y}$ with $\bar{y}_m$ being the maximum value of the measured $\bar{y}$ values.

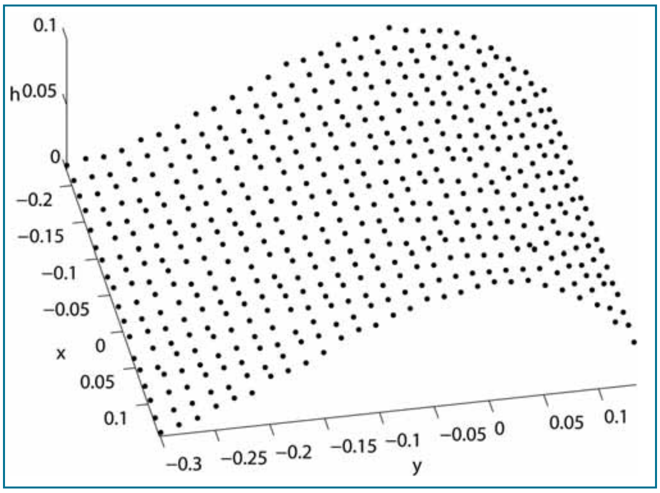

Fig. 2: Measured points of a $18 \times 28 \times 7$ spatial grid at pressure $p_s = 7$

The measurements of the coordinates of the grid of 18×28 points at pressure $p_s = 1$ are repeated $n_w = 20$ times. The standard deviations σ_x, σ_y and σ_h of the coordinates obtained by (29) are

$$\sigma_x = 0.11\text{mm}, \; \sigma_y = 0.15\text{mm}, \; \sigma_h = 1.81\text{mm} \tag{30}$$

and the standard deviation from (23) of positioning one point is

$$\sigma_p = 1.82\text{mm}. \tag{31}$$

Fig. 1 and 2 give the impression that the coordinates of the points have not been measured for a regular grid. This is due to the perspective view of the variations of the measured heights h caused by the variances. This shows up especially in Fig. 3, where the measured points at pressure $p_s \in \{1, 3, 5, 7\}$ are drawn.

The three-dimensional B-spline surface (1) is fitted by the lofting method to the measured coordinates of the spatial grid of $18 \times 28 \times 7$ points for the seven states of pressure p_s. To find a good fit measured by the standard deviations σ_x, σ_y and σ_h of the coordinates from (22) and

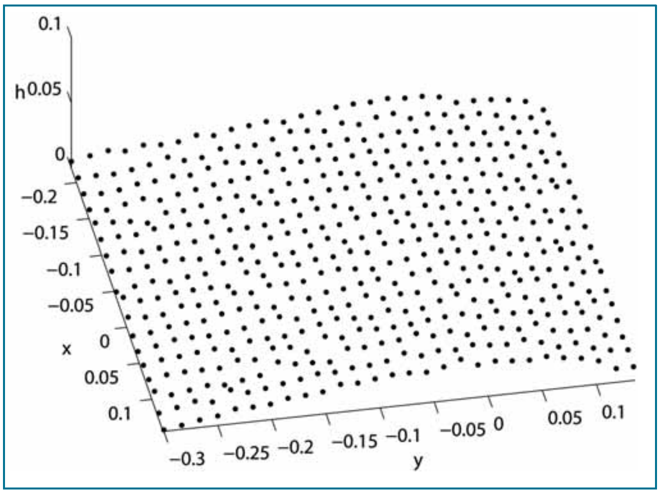

Fig. 1: Measured points of a $18 \times 28 \times 7$ spatial grid at pressure $p_s = 1$

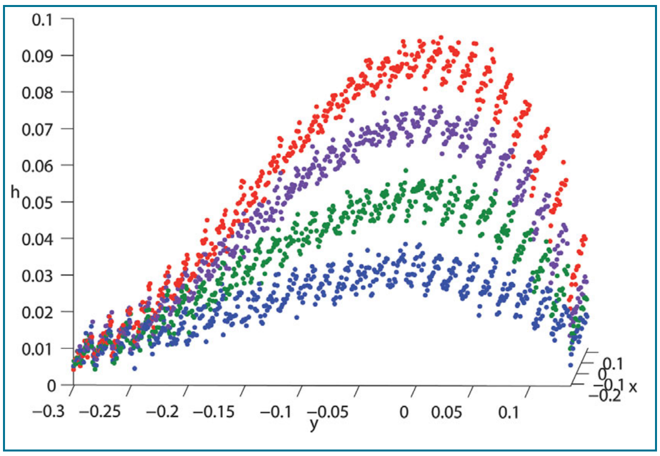

Fig. 3: Measured points at $p_s = 1$ in blue, at $p_s = 3$ in green, at $p_s = 5$ in violet and at $p_s = 7$ in red

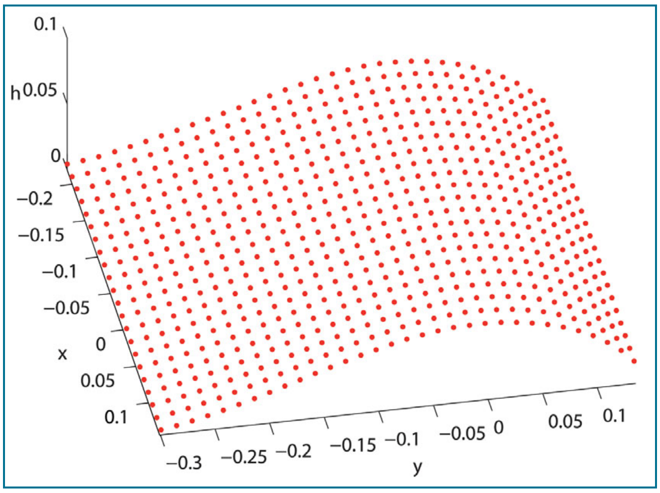

Fig. 4: Computed points of a 22 × 32 × 9 spatial grid on the surface at pressure $p_s = 4$

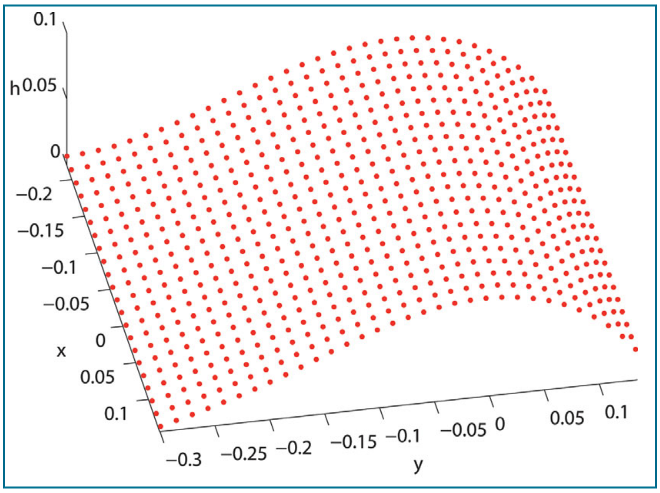

Fig. 5: Computed points of a 22 × 32 × 9 spatial grid on the surface at pressure $p_s = 5.5$

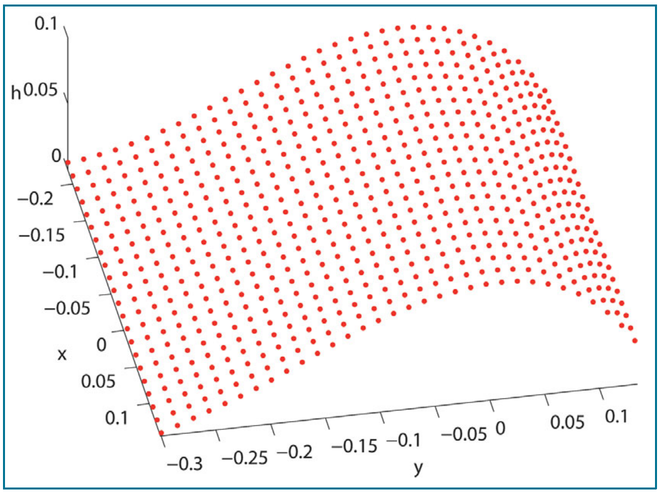

Fig. 6: Computed points of a 22 × 32 × 9 spatial grid on the surface at pressure $p_s = 7$

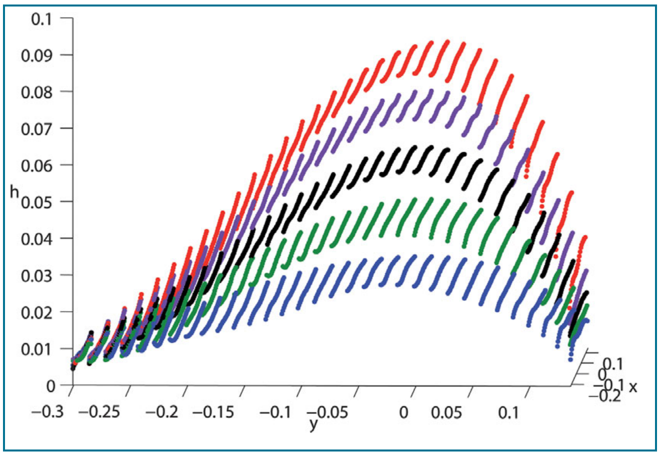

Fig. 7: Computed points at $p_s = 1$ in blue, at $p_s = 2.5$ in green, at $p_s = 4$ in black, at $p_s = 5.5$ in violet and at $p_s = 7$ in red

by the standard deviation σ_p of positioning a point from (23), the degrees $p = 2$ and $q = r = 3$ have been chosen in (1). Furthermore, small standard deviations were searched for a minimum number of polynomial segments for the isoparametric curves in the direction of x, y and p_s. They are found by (22) with

$$\sigma_x = 0.16\text{mm},\ \sigma_y = 0.15\text{mm},\ \sigma_h = 1.81\text{mm} \tag{32}$$

and by (23) with

$$\sigma_p = 1.83\text{mm}. \tag{33}$$

These standard deviations agree very well with the standard deviations of (30) and (31). The B-spline surface is therefore fitted to the measurements with an accuracy which agrees with the accuracy of the measurements. Modeling errors caused by the fit of the B-spline surface do not occur.

The B-spline surface does not oscillate between the measured points, because $l + 1 = 5, m + 1 = 8$ and $n + 1 = 5$ are chosen for (1) which results in 3 polynomial segments for the isoparametric curves in the direction of x, 5 segments in the direction of y and 2 segments in the direction of p_s. The 3 polynomial segments in the direction of x are determined by 18 points, the 5 segments in the direction of y by 28 points and the 2 segments in the direction of p_s by 7 points. This results in a smooth surface as can be seen by Fig. 4 to Fig. 6. They show a spatial grid of 22 × 32 × 9 points for $p_s \in \{4, 5.5, 7\}$ computed by (24) to (26) on the surface of the estimated B-spline surface. This grid does not coincide with the grid of points of the measurements. The points at pressure $p_s = 5.5$, for which no measurements have been taken, fit smoothly between the points at pressure $p_s = 4$ and $p_s = 7$. This can also be very well recognized in Fig. 7, where the computed points at pressure $p_s \in \{1, 2.5, 4, 5.5, 7\}$ are shown.

6 Conclusions

The three-dimensional B-spline surface is fitted to the coordinates, measured by a laserscanner, of points on a thin plastic sheet. The sheet is bent under pressure and the coordinates are measured for different values of the pressure. Points on the deformed surface are computed for values of the pressure which lie between the values of the pressure for the measurements. They provide a smoothly varying shape of the B-spline surface. It is shown that the B-spline surface is fitted to the measured coordinates with an accuracy which agrees very well with the accuracy of the measurements.

References

[1] Cox, M. G. (1972): The numerical evaluation of B-splines. J Institute of Mathematical Applications, 10: 134–149

[2] Boor, C. de (1972): On calculating with B-splines. J Approximation Theory, 6: 50–62

[3] Farin, G. E.; Hansford, D. (2000): The Essentials of CAGD. A K Peters, Natick

[4] Hennes, M. (2009): Freiformflächenerfassung mit Lasertrackern – eine ergonomische Softwarelösung zur Reflektoroffsetkorrektur. Allgemeine Vermessungs-Nachrichten, 116: 188–194

[5] Koch, K. R. (1999): Parameter Estimation and Hypothesis Testing in Linear Models, 2nd Ed. Springer, Berlin

[6] Koch, K. R. (2007): Introduction to Bayesian Statistics, 2nd Ed. Springer, Berlin

[7] Koch, K. R. (2008): Determining uncertainties of correlated measurements by Monte Carlo simulations applied to laserscanning. J Applied Geodesy, 2: 139–147

[8] Koch, K. R. (2009a): Fitting free-form surfaces to laserscan data by NURBS. Allgemeine Vermessungs-Nachrichten, 116: 134–140

[9] Koch, K. R. (2009b): Identity of simultaneous estimates of control points and of their estimates by the lofting method for NURBS surface fitting. Int J Advanced Manufacturing Technology, 44: 1175–1180

[10] Koch, K. R. (2009c): Uncertainty of NURBS surface fit by Monte Carlo simulations. J Applied Geodesy, 3: 239–247

[11] Koch, K. R. (2010): Three-dimensional NURBS surface estimated by the lofting method. Int J Advanced Manufacturing Technology, DOI 10.1007/s00170-009-2460-6

[12] Ma, W.; Kruth, J.-P. (1998): NURBS curve and surface fitting for reverse engineering. Int J Advanced Manufacturing Technology, 14: 918–927

[13] Park, H. (2001): An approximate lofting approach for B-spline surface fitting to functional surfaces. Int J Advanced Manufacturing Technology, 18: 474–482

[14] Piegl, L. (1991): On NURBS: a survey. IEEE Computer Graphics and Applications, 10: 55–71

[15] Piegl, L.; Tiller, W. (1997) The NURBS Book, 2nd Ed. Springer, Berlin

[16] Rogers, D. F. (2001): An Introduction to NURBS. Academic Press, San Diego

[17] Schmidt, M. (2007): Wavelet modelling in support of IRI. J Adv Space Res, 39: 932–940

[18] Tiller, W. (1983): Rational B-splines for curve and surface representation. IEEE Computer Graphics and Applications, 4: 61–69

[19] Varady, T.; Martin, R. R.; Cox, J. (1997): Reverse engineering of geometric models – an introduction. Computer-Aided Design, 29: 255–268

[20] Zeilhofer, C.; Schmidt, M.; Bilitza, D.; Shum, C. K. (2009): Regional 4-D modeling of the ionospheric electron density from satellite data and IRI. J Adv Space Res, 43: 1669–1675

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Author's adress:
Prof. Dr.-Ing., Dr.-Ing. E.h.mult. Karl Rudolf Koch (em.)
Institute for Geodesy and Geoinformation, Theoretical Geodesy, University of Bonn, Nussallee 17, D-53115 Bonn, Germany, email: koch@geod.uni-bonn.de

Abstract

Fitting free-form surfaces to measured coordinates of points on the surface of an object is generally solved by the tensor product of NURBS (nonuniform rational B-splines). NURBS surfaces are needed in reverse engineering and also in engineering geodesy. The NURBS surface is generalized here so that a surface with a changing shape is obtained. It can be applied in deformation analysis if the shape of the deformed surface is needed between the time epochs of the measurements. Surfaces can also be represented which depend on points with three-dimensional coordinates. The coordinates of points on the surface of a thin plastic sheet are measured by the laserscanner Leica HDS 3000. The sheet is bent under increasing pressure, and the coordinates are measured for different values of the pressure. The generalized NURBS surface is fitted to the measurements by estimating cross-sectional curves. The points on the deformed surface are computed for values of the pressure which lie between the values for the measurements.