Application of a damping function in adjustment of GPS networks with long vectors

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The aim of this study is to examine the effect of long GPS vectors on the precision of determination in monitoring terrain dislocation. The results will be processed numerically with the classical method of least squares and with selected damping functions [5, 6]. In the latter case, selection of the appropriate criteria of the damping function will be an important issue [7]. The study is based on the actual results of measurements performed in a landslide in Siercza near Kraków. Apart from the measurements by the static GPS method, classical situational and height measurements were performed for comparison. The results are taken from one periodic measurement (cycle). The comparative analyses are based on the results of adjustment of satellite (GPS), classical (on-the-ground) and integrated (classical measurements with GPS observations) networks.

1 Introduction

Growing popularity is recently enjoyed by static GPS measurements, based on the network of reference points of the EUPOS system [3, 8, 14]. The main advantage of the system (from a user's point of view) is that the receiver does not have to be placed at a fixed (reference) point. As a result, in order to perform a measurement with reference to a fixed coordinate system, it is enough to have one GPS receiver and one does need to worry about access to points of national control network. However, too great distance from the object to reference points (even up to 100 km) may be a fundamental drawback of such a solution. It is well known that long vectors bear considerable errors, both in terms of their length and their orientation in space [13]. Measurement errors in turn determine the accuracy of establishing the position (mean error of a point position) and they indirectly affect the precision of values determined as the functions of point coordinates (e.g. length). The precision of determination achieved in measurements may in consequence become a decisive factor, necessitating the selection of another measurement method which meets the adopted requirements, even despite increased cost and amount of work necessary to perform the field work. Such restrictive requirements regarding the precision exist, e.g. in geodetic determination of distortion and dislocation of objects [15]. This relates both to absolute (position) and relative (distance between points) dislocations of points. In geodetic tasks of this type a situation often occurs that it is not possible to refer the local measurement control network, representing the dislocation object, with an external, fixed coordinate system. The reason is usually simple: in the vicinity of the measurement object "everything is moving" (e.g. in the area of mining exploitation or in a vast landslide). However, a fixed position (invariability) of an external system of reference is recommended in dislocation studies during the time necessary to perform many series (cycles) of the planned periodical measurements [2, 15]. And here we arrive at a situation when using distant reference points is highly desirable.
Is there a solution of the problem? It seems that too much emphasis is put on the measurement and result processing is usually performed with the classical least square method. On the other hand, methods of strong (robust) estimation (e.g. [11, 18]) are still underestimated; their task is not only to eliminate the observations with gross errors, but also to reduce the effect of outlying observations.

2 Characteristics of the study object and the test measurements performed

Simultaneously with the periodical test measurements, geological studies were conducted in the landslide, aimed at finding the reasons of the long-term earth movements which pose a threat to people living nearby. The area was monitored by the Kraków Geological Company on the order from the District Governor's Office in Wieliczka (near Kraków). Stabilisation of the point base and performing the measurements for this study were possible after the relevant permits and the landslide documentation were obtained from those institutions. The measurement base was established at the place where the most massive movements of earth took place, near the local road and the residential buildings. The relief of the area is highly diverse, with a lot of slopes and micro-slides. Height differences reach 20 m along the distance of 60 m. The task was made even more difficult by trees, shrubs and property fences. Performing the geodetic measurements in such conditions, both classical and GPS ones, required careful preparation (planning) for each method.

2.1 GPS measurement (static method)

The aim of the measurement was to determine the mutual position of the object points, which was ensured by the vectors obtained in various combinations between the points (Fig. 2). Additionally, the determined points were referred to the global reference system WGS84 with six available reference points of the EUPOS system. The points were necessary for later postprocessing and transformation of the coordinates onto the ellipsoidal system GRS80. The satellite signals were simultaneously recorded by four GePos RM24 receivers, manufactured by *Carl Zeiss*. During a measurement series, the receivers stayed at the points for 60 minutes.

2.2 Modular situational (horizontal) network

In this case, the established points generally played the role of so called aiming points (Fig. 3), and the measuring sites (stations) were chosen in convenient places – without stabilisation (these are typical features of so called modular networks [4]). In order to perform a measurement at a specific site it was necessary to place several tripods with levelling heads and prisms above the established points.

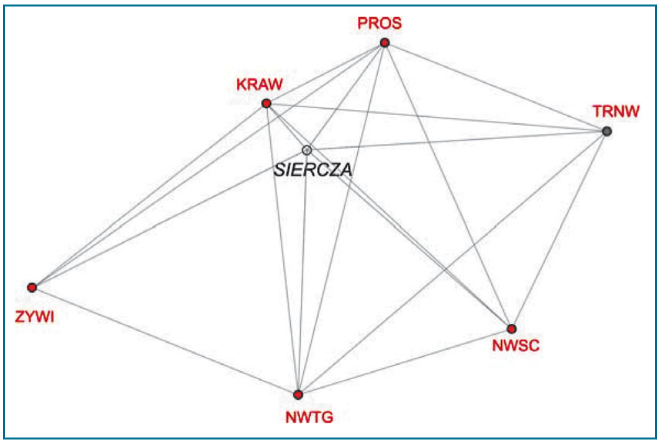


***Fig. 1:** GPS measurement – referring the object with the reference points of the EUPOS system*

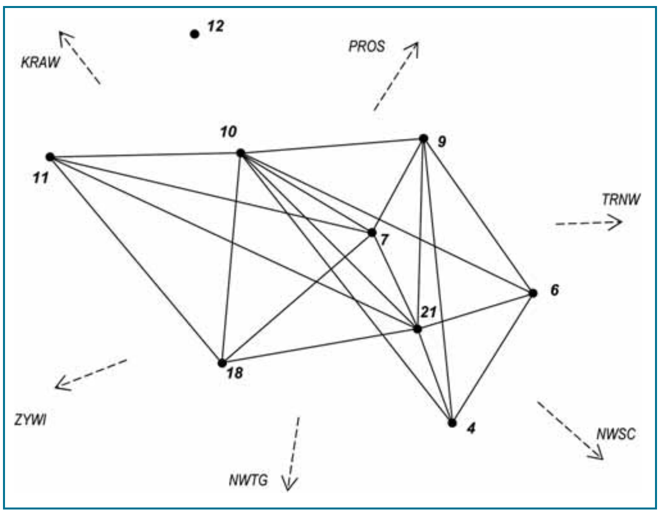


***Fig. 2:** GPS measurement (local vectors); no referring to point no. 12 due to unfavourable exposure to the satellite signals*

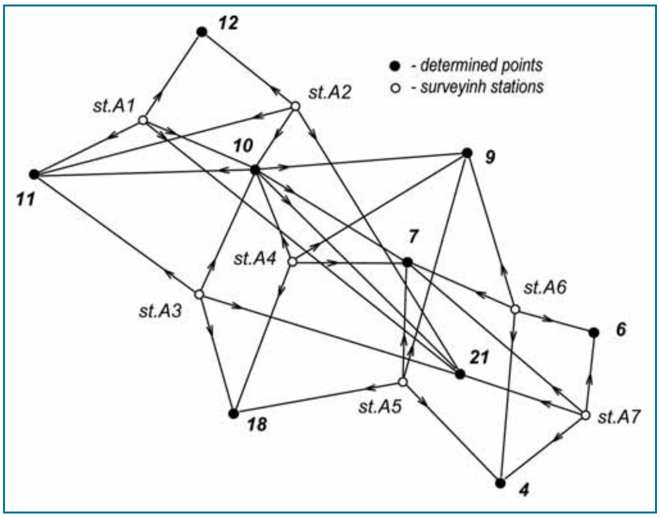


***Fig. 3:** Measurement by the modular network method*

The measuring sites were of the lost (temporary) point type, so in consequence the instrument did not need to be centred. The modular network method proved to be the only feasible form of plane and height measurement (along with the GPS method). The classical plane and height measurement (centred measuring sites) was given up due to great difficulties in achieving visibility between the established points.

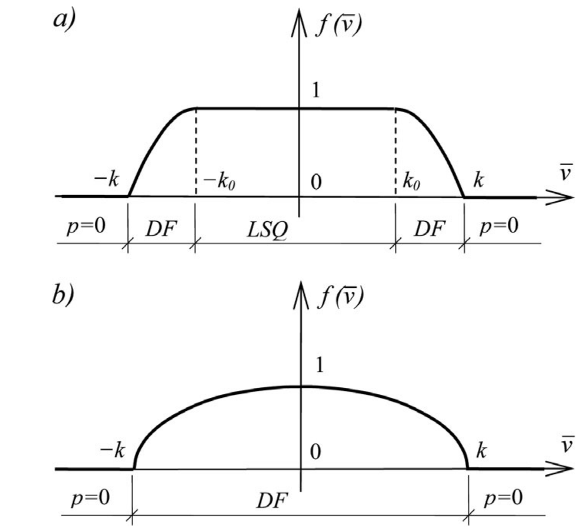
2.3 Geometrical levelling

Relative altitudes (local reference system) were determined based on the results of the levelling network measurement. Altitude differences were determined four times (twice in each direction). The method of precise geometrical levelling was applied, with the use of a Leica NA3003 digital levelling instrument with invar code staves. The measurement results were recorded automatically in the instrument's memory carrier.

3 The damping functions applied in the experiment

The least square method is a standard way of adjustment of geodetic measurement results; it treats all observations identically, even those with gross errors [17]. The aim of the damping function is to compensate (damp) the effect of outlying observations (gross errors) on the final results of adjustment [15]. Among the known damping functions, there are the following: Huber's function [10], Hampel's function [9], Danish function [17], as well 2 functions proposed by the author of this paper, conventionally marked as *QDF* [5] and *EDF* [6] (cf. Fig. 4).
The principles of application of the damping functions, presented in Fig, are as follows:
a) *QDF* (see [5])

$$f(\bar{v}) = \begin{cases} 1; & \bar{v} \in \langle -k_0; k_0 \rangle \\ 1 - \frac{\bar{v}^2 - 2k_0|\bar{v}| + k_0^2}{(k-k_0)^2}; & |\bar{v}| \in \langle k_0; k \rangle \\ 0; & |\bar{v}| > k \end{cases} \qquad (1)$$



Notation:
$\bar{v}$ – standardised adjustment; $f(\bar{v})$ damping function;
k_0, k – damping function criteria; *LSQ* – least square method;
DF – application of a damping function;
$p = 0$ – rejection of an observation (p – observation weight)

***Fig. 4:** The damping functions applied in the experiment: **a)** QDF, **b)** EDF*

b) *EDF* (see. [6])

$$f(\bar{v}) = \begin{cases} 1; & \bar{v} = 0 \\ \sqrt{1 - \frac{\bar{v}^2}{k^2}}; & \bar{v} \in \langle -k; k \rangle \\ 0; & |\bar{v}| > k \end{cases} \tag{2}$$

Notation of symbols in formulae (1) and (2) are consistent with Fig. 4.

4 Standardisation of corrections and establishing the damping function criteria

Two methods (for comparison) of standardisation of corrections of the least square method were applied.
The traditional method of standardisation (according to [5, 7])
Standardisation is performed based on the original correction values v_i, obtained by the classical LSQ:

$$\bar{v}_i = \frac{v_i}{\sqrt{Q_{ii}}} \tag{3}$$

where:
Q_{ii} – diagonal elements of the covariance matrix $\mathbf{Qv}$ of the corrections vector $\mathbf{V}$, estimated by *LSQ*:

$$\mathbf{Q_V} = \mathbf{P}^{-1} - \mathbf{A}(\mathbf{A}^T\mathbf{PA})^{-1}\mathbf{A}^T \tag{4}$$

($\mathbf{A}$ – matrix of factors at the unknowns; $\mathbf{P}$ – matrix of weights).
The parameter k_0, which is present in equation (1), is selected according to the normal distribution of observation errors. It is necessary to determine the level of probability γ, with which the standardised correction should lie within the interval $\langle -k_0;\ k_0 \rangle$:

$$P(\bar{v} \in \langle -k_0; k_0 \rangle) = 2 \int_0^{k_0} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-\bar{v}^2}{2}\right) d\bar{v} = 2\phi(k_0) = \gamma \tag{5}$$

The value of function $\phi(k_0)$ is taken from the tables of normal distribution for argument k_0. The criterion of the damping function k (equations (1), (2)) is adopted empirically [5].
The simplified method of standardisation (according to [7]) Standardised corrections ($v_i \rightarrow \bar{v}_i$):

$$\bar{v}_i = \frac{v_i}{m_i} \tag{6}$$

where: m_i – mean error of the i-th observation.
Criteria of the damping function (k, k_0):

$$k = 1.5 \cdot |\bar{v}|_{ar} \tag{7}$$

$$k_0 = 0.5 \cdot |\bar{v}|_{ar} \tag{8}$$

$$|\bar{v}|_{ar} = \frac{\sum |\bar{v}_i|}{n} \tag{9}$$

where: $|\bar{v}|_{ar}$ – arithmetic average of the absolute values of standardised corrections; n – the number of observations in a group of uniform observations.

5 Numerical processing of the measurement results and the comparative analysis

Results of measurements performed by the GPS technique (Fig. 1, 2) were processed according to several variants (Table 1), depending on the manner in which the measurement point network was referred to the external reference system (EUPOS). The numerical processing (adjustment) of the network was performed with the computational system GEONET [12]. As the classical (ground) measurements were performed in a free system (no reference), the precision of determination of the absolute position with the two techniques (GPS and classical method) cannot be directly compared. Consequently, an integrated network was developed as a comparative variant, the network consisting of GPS vectors and classical observations: distances and horizontal angles as well as altitude differences from geometrical levelling.
The integrated network was developed in the following stages (cf. [13]):

1) Adjustment of GPS measurements in the ellipsoid system GRS80 (ETRF-89) and converting the Cartesian coordinates *XYZ* to ellipsoidal coordinates *BLH*.
2) Projection of GPS vectors onto the ellipsoid, i.e. changing each vector (ΔX, ΔY, ΔZ) into so called pseudo-observations: initial azimuth of the geodetic line A and its length s and the difference of ellipsoidal altitudes ΔH

$$(\Delta X,\ \Delta Y,\ \Delta Z) \rightarrow (A,\ s,\ \Delta H) \tag{10}$$

3) Correction of classic lengths d, assuming that projection is performed onto the geoid and onto the ellipsoid

$$\delta d = \frac{(H_{sr} + N_{sr}) \cdot d}{R} \tag{11}$$

H_{sr} – average normal altitude of the terminal points of section d,
N_{sr} – mean distance between the geoid and the ellipsoid GRS80 ($\sim$ 34 m in the territory of Poland).

4) Creating a set of observations (pseudo-observations) on the ellipsoid, consisting of: classical lengths d and horizontal angles as well as azimuths A and lengths of geodetic lines s.
5) Adjustment of the integrated (2-dimensional) network on the ellipsoid and calculation of the adjusted ellipsoidal coordinates $B^{(\mathrm{w})}$, $L^{(\mathrm{w})}$ (the $^{(\mathrm{w})}$ symbol denotes the adjusted values).
6) Transformation of ellipsoidal differences of altitude (pseudo-observations ΔH) into ordinary differences of altitudes ΔH_n.

$$\Delta H_n = \Delta H - \Delta N \tag{12}$$

ΔN – the difference of distances between the geoid and the ellipsoid (based on the numerical model of the geoid).

7) Preparation of a set of normal altitude differences, consisting of satellite measurements ΔH_n and classical observations from geometrical levelling.
8) Adjustment of integrated levelling; calculation of adjusted normal altitudes $H_n^{(\mathrm{w})}$.
9) Transformation of normal altitudes $H_n^{(\mathrm{w})}$ into ellipsoidal altitudes $H^{(\mathrm{w})}$

$$H^{(\mathrm{w})} = H_n^{(\mathrm{w})} + N \tag{13}$$

N – distance between the geoid and the ellipsoid (based on the numerical model of the geoid).

10) Transformation of ellipsoidal coordinates $B^{(\mathrm{w})} L^{(\mathrm{w})} H^{(\mathrm{w})}$ into geocentric coordinates $X^{(\mathrm{w})} Y^{(\mathrm{w})} Z^{(\mathrm{w})}$.

Calculations of type (10) are based on the basic formulae of higher geodesy [13]. Corrections for classical lengths (11) are important in large height differences (of the order of dozens of metres) and distances of several hundred metres. In our site the height differences were no greater than 20 m and the distances were shorter than 100 m. Formula (11) should not be referred to geodetic lines as these are the lengths already reduced onto the ellipsoid (10). Transformations of height differences to normal ones (12) resulted in their change within the range $0 \div 3$ mm. The reverse transformation (onto the ellipsoid) (13) was used to convert the geodetic coordinates into Cartesian ones.

The computational issues related to integrated networks, including their application in monitoring dislocations and distortions, have been dealt with in numerous studies (e.g. [1, 2, 4, 16]).

Table 1 specifies the adjusted coordinates of an integrated network and deviations of a GPS network which has been adjusted in several example versions. In theory, the *GPS-1* version is the most precise (of the GPS networks) due to beneficial reference (incidences) of point 10 with other points (see Fig. 2) and due to good exposure to satellite signals. The average resultant deviation of coordinates (lower line of table 1) can be used as the global comparative parameter for particular versions. Apparently, the discrepancies in the results may be important in terms of the accuracy required in determination of absolute dislocations.

The coordinates (positions of points) also affect the distances between the determined points, which in turn are of key importance in studying relative (mutual) dislocations. Table 2 specifies the lengths from classical (more precise) measurements and deviations from the lengths calculated after adjusting the integrated network. When analysing the results, attention should be paid to the differences between particular versions (*GPS-1*, *GPS-2*, *GPS-3*, *GPS-4*) rather than to the values of the deviations. The differences are rather small when compared to the differences of coordinates (Table 1). This may indicate the existence of a certain systematic error which similarly affects the accuracy of all the GPS vectors.

Table 1: Deviations of coordinates for different variants of adjustment of GPS networks.

Pt. No.	Coordinates (WGS84)			Deviations for individual variants of network adjustment															
	GPS-CLASS			*GPS-1*				*GPS-2*				*GPS-3*				*GPS-4*			
	X	*Y*	*Z*	*dX*	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*
4	3861268.077	1409139.258	4861199.465	– 0.006	0.005	– 0.009	0.007	0.001	0.000	– 0.010	0.006	– 0.004	0.003	– 0.017	0.010	0.016	– 0.001	0.014	0.012
6	3861237.261	1409147.429	4861216.119	– 0.012	– 0.004	– 0.003	0.008	– 0.011	– 0.010	– 0.004	0.009	– 0.012	– 0.006	– 0.013	0.011	0.009	– 0.010	0.021	0.015
7	3861236.417	1409103.952	4861219.381	0.019	0.010	0.012	0.014	0.035	0.002	0.019	0.023	0.020	0.009	0.004	0.013	0.040	0.002	0.035	0.031
9	3861215.216	1409107.512	4861232.745	– 0.007	0.020	0.006	0.013	0.008	0.025	0.028	0.022	– 0.005	0.019	0.000	0.012	0.018	0.018	0.036	0.025
10	3861234.468	1409068.595	4861230.800	– 0.001	0.000	0.000	0.001	0.003	– 0.002	0.004	0.003	0.001	– 0.002	– 0.006	0.004	0.021	– 0.007	0.024	0.019
11	3861254.000	1409025.277	4861230.630	– 0.015	0.009	– 0.006	0.011	– 0.003	0.010	0.012	0.009	– 0.016	0.006	– 0.014	0.013	0.016	– 0.004	0.025	0.017
18	3861276.045	1409079.873	4861202.647	0.002	0.008	– 0.002	0.005	– 0.002	0.010	0.007	0.007	0.003	0.006	– 0.008	0.006	0.020	– 0.005	0.018	0.016
21	3861252.411	1409123.062	4861208.445	– 0.004	0.000	0.002	0.003	– 0.001	– 0.006	0.006	0.005	– 0.003	– 0.002	– 0.006	0.004	0.017	– 0.007	0.026	0.018
–	–	–	–	Average			0.0075	Average			0.0106	Average			0.0091	Average			0.0191

Notation:
GPS-CLASS – Integrated network referred only to point No. 10; *GPS-1* – GPS network referred only to point No. 10; *GPS-2* – GPS network entirely (referred to all points); *GPS-3* – GPS network with one reference point *KRAW*; *GPS-3* – GPS network with one reference point *KRAW*; *dX*, *dY*, *dZ*, *dL* – deviations of coordinates and resultant deviation (relative to integrated network).

The variability of accuracy of the measured GPS vectors is illustrated by the data contained in Table 3. It specifies 18 examples of randomly selected vectors (of all the 79 observed vectors) with the mean errors of their measurement (standard deviations). The last two columns also specify the resultant mean errors σL and vector lengths L. It can be easily noticed that for short (local) vectors, the errors assume the values of several centimetres (5 cm max – for the vector not mentioned in table 3). The measurement errors for long reference vectors (over 10 km) are usually close to several decimetres, with their maximum values exceeding 1 m. In such cases the occurrence of gross errors or outlying observations may be considered (cf. [11, 17]).

The effectiveness of compensating the errors of GPS vectors was analysed on the *GPS-2* version network (see ta-

Table 2: Deviations of the lengths of GPS vectors in relation to the lengths from classical measurements

Vector		Length	Deviations				
from	to	*CLASS*	*GPS-CLASS*	*GPS-1*	*GPS-2*	*GPS-3*	*GPS-4*
11	10	47.513	0.005	– 0.010	– 0.009	– 0.009	0.001
11	21	100.272	0.010	0.000	– 0.004	0.000	0.007
11	7	81.388	0.009	0.000	– 0.008	0.001	0.009
18	10	51.456	0.006	0.011	0.003	0.011	0.009
18	11	65.182	0.007	0.010	0.010	0.011	0.011
18	21	49.567	0.005	0.002	– 0.009	0.002	0.005
18	7	49.292	0.006	– 0.002	– 0.024	– 0.002	– 0.002
21	10	61.543	0.006	0.005	0.001	0.005	0.004
4	10	84.280	0.009	0.015	0.015	0.015	0.016
4	21	24.254	0.003	0.009	0.014	0.009	0.011
4	6	35.965	0.004	0.011	0.015	0.010	0.011
6	10	80.229	0.008	0.005	0.001	0.005	0.006
6	21	29.698	0.003	0.003	0.002	0.003	0.004
7	10	37.202	0.004	0.012	0.006	0.012	0.011
7	21	27.211	0.003	– 0.014	– 0.019	– 0.014	– 0.014
9	10	43.458	0.005	0.026	0.028	0.027	0.029
9	21	47.067	0.005	0.003	0.000	0.003	0.002
9	7	25.310	0.003	0.023	0.034	0.024	0.025
Average			0.0056	0.0090	0.0112	0.0091	0.0097
Max			0.0103	0.0261	0.0339	0.0270	0.0291

Notation:
CLASS – network of classical (terrestrial) survey

Table 3: Examples of GPS observations with mean errors

Vector		Components of vector			Mean errors				Length
from	to	ΔX	ΔY	ΔZ	σx	σy	σz	σL	L
11	10	– 19.519	43.307	0.175	0.008	0.008	0.011	**0.016**	47.503
18	11	– 22.058	– 54.594	27.980	0.010	0.009	0.014	**0.019**	65.192
4	9	– 52.857	– 31.728	33.310	0.012	0.011	0.020	**0.026**	70.072
9	21	37.197	15.528	– 24.299	0.012	0.010	0.014	**0.020**	47.066
9	6	22.035	39.892	– 16.645	0.011	0.010	0.016	**0.022**	48.517
KRAW	10	4 298.305	11 318.119	– 6 488.639	0.014	0.013	0.015	**0.025**	13 735.999
KRAW	11	4 317.698	11 274.775	– 6 488.865	0.053	0.036	0.037	**0.074**	13 706.511
NWSC	10	– 12 227.285	– 53 819.552	25 574.787	0.010	0.008	0.010	**0.016**	60 828.615
NWSC	6	– 12 224.466	– 53 740.639	25 560.131	0.323	0.443	0.288	**0.619**	60 752.071
NWTG	21	– 39 798.465	– 13 249.996	35 176.159	0.011	0.010	0.013	**0.019**	54 743.423
NWTG	6	– 39 813.606	– 13 225.544	35 183.842	0.318	0.427	0.280	**0.601**	54 753.457
PROS	11	23 425.027	– 9 754.616	– 15 455.002	0.660	0.822	0.596	**1.211**	29 710.965
PROS	ZYWI	66 804.324	– 58 588.002	– 36 054.795	0.021	0.011	0.026	**0.035**	95 892.231
TRNW	4	26 952.314	– 61 499.098	– 2 951.313	0.009	0.008	0.016	**0.020**	67 210.688
TRNW	6	26 921.507	– 61 490.859	– 2 934.635	0.279	0.372	0.242	**0.524**	67 190.069
TRNW	KRAW	22 620.430	– 72 887.838	3 568.696	0.208	0.206	0.204	**0.357**	76 400.631
ZYWI	18	– 43 357.360	48 887.985	20 571.825	0.407	0.552	0.369	**0.778**	68 506.173
ZYWI	7	– 43 396.847	48 911.888	20 588.463	0.033	0.029	0.039	**0.059**	68 553.220

Notation:
σL – resultant of mean errors of vector components

Table 4: Standardisation of adjustment corrections

Vector		*LSQ* metod (no standardisation)			Traditional standardisation			Simplified standardisation		
from	to	v_x	v_y	v_z	v_x	v_y	v_z	v_x	v_y	v_z
11	10	– 0.007	– 0.001	– 0.013	– 0.55	– 0.08	– 0.72	– 0.94	– 0.14	– 1.13
18	11	– 0.014	0.002	– 0.008	– 0.83	0.10	– 0.38	– 1.34	0.16	– 0.58
4	9	0.003	0.007	0.010	0.13	0.38	0.31	0.21	0.65	0.49
9	21	0.010	0.010	0.024	0.51	0.56	1.08	0.86	1.00	1.72
9	6	0.008	0.010	0.013	0.43	0.58	0.53	0.73	1.01	0.84
KRAW	10	0.008	0.008	0.006	0.33	0.32	0.24	0.58	0.58	0.42
KRAW	11	– 0.125	– 0.030	– 0.058	– 1.45	– 0.52	– 0.97	– 2.36	– 0.84	– 1.58
NWSC	10	0.012	0.007	0.006	0.65	0.43	0.32	1.25	0.87	0.60
NWSC	6	– 0.051	– 0.095	– 0.038	– 0.10	– 0.13	– 0.08	– 0.16	– 0.21	– 0.13
NWTG	21	0.011	– 0.004	0.015	0.52	– 0.19	0.66	0.96	– 0.37	1.17
NWTG	6	– 0.029	– 0.085	– 0.033	– 0.06	– 0.12	– 0.07	– 0.09	– 0.20	– 0.12
PROS	11	0.049	0.008	– 0.030	0.05	0.01	– 0.03	0.07	0.01	– 0.05
PROS	ZYWI	– 0.023	– 0.017	– 0.028	– 0.64	– 0.83	– 0.63	– 1.10	– 1.61	– 1.06
TRNW	4	– 0.001	0.005	0.024	– 0.08	0.28	0.86	– 0.15	0.59	1.46
TRNW	6	– 0.022	– 0.074	0.007	– 0.05	– 0.12	0.02	– 0.08	– 0.20	0.03
TRNW	KRAW	0.021	0.032	– 0.009	0.06	0.09	– 0.03	0.10	0.15	– 0.05
ZYWI	18	– 0.082	– 0.005	– 0.039	– 0.12	– 0.01	– 0.06	– 0.20	– 0.01	– 0.11
ZYWI	7	0.022	– 0.174	– 0.148	0.40	– 3.69	– 2.36	0.65	– 6.01	– 3.83

ble 1), i.e. one that contains the whole set – 79 of the observed vectors (see Fig. 1, 2) Two independent damping functions were applied: *QDF* (1) and *EDF* (2). Two methods of standardisation of observation corrections and establishing the damping function criteria were tested: the traditional method – formulae (3) to (5) – and a simplified method – formulae (6) to (9). At the first stage of calculations the network was adjusted in accordance with the classical LSQ method. The observation corrections and the standardised corrections are shown in table 4 (for selected vectors – like in Table 3).

Standardised corrections (Table 4), mean errors of corrections (calculated in accordance with (4)) and mean errors of observations (Table 3) were the basis of determination of the damping function for particular components of each vector (Table 5).

The criteria of the damping function were established in the manner described below.

a) Criteria applied in traditional standardisation:

– the level of probability was adopted to be $\gamma = 0.8$,
– for function $\phi(k_0) = 0.4$ (cf. formula (5)), the value of parameter $k_0 = 1.3$ was taken from the tables of normal distribution,
– the auxiliary criterion k was adopted as twice the value of k_0, i.e. $k = 2.6$.

b) Criteria applied in simplified standardisation:

– mean value of the standardised corrections $|\bar{v}|_{ar} = 1.05$ (according to (9)),
– criterion $k_0 = 0.5$ (according to (8)),
– auxiliary criterion $k = 1.6$ (according to (7)).

Table 4 contains examples of the damping function for 4 different versions (combinations of two damping functions and two methods of standardisation). It shows that the value of the damping function is related to the mean error of observation. The *EDF* type functions dampen outlying observations in a mild way, whereas *EDF* functions do it in a more decisive way (cf. Fig. 4). The value of 0.0001 is numerically equal to zero – the observation is excluded from the adjustment process. It is noteworthy that in such a case it is not necessarily the entire vector that is excluded, but only its specific component. The values of the damping function are used at the next stage as weight multipliers, obtained from the classical LSQ method. Thus modified, the weights provide a basis for another adjustment of the observation. Table 6 contains deviations from the GPS-2 version network coordinates adjusted in this way, in relation to the integrated network version, like in comparison of the original observations – cf. Table 1). Such juxtaposition was made in order to check whether the tendency of changes resulting from application of the damping function is correct – the deviations are getting smaller. When analysing the mean values of the resultant deviations (the penultimate line of the table), it may be noticed that the decisive role in the damping process is played not by the type of the function applied, but by the method of standardisation of corrections and establishing the damping criteria. The second – simplified – method of standardisation is more beneficial from the point of view of the purpose to be achieved; in the final effect it allows for ca. 20 % reduction of the coordinate errors (see the lower line in table 6). A better effect can be achieved by empirical selection of the appropriate values of the damping function criteria, especially its parameter k (it will be the subject of further studies conducted

Table 5: The values of the damping function according to different variants

Vector		Mean errors			*KTF-trad*			*ETF-trad*			*KTF-smpl*			*ETF-smpl*		
from	to	σx	σy	σz	$f(x)$	$f(y)$	$f(z)$	$f(x)$	$f(y)$	$f(z)$	$f(x)$	$f(y)$	$f(z)$	$f(x)$	$f(y)$	$f(z)$
11	10	0.008	0.008	0.011	1	1	1	0.98	1	0.96	0.84	1	0.67	0.81	1	0.71
18	11	0.010	0.009	0.014	1	1	1	0.95	1	0.99	0.42	1	0.99	0.55	1	0.93
4	9	0.012	0.011	0.020	1	1	1	1	0.99	0.99	1	0.98	1	0.99	0.91	0.95
9	21	0.012	0.010	0.014	1	1	1	0.98	0.98	0.91	0.89	0.79	0.0001	0.84	0.78	0.0001
9	6	0.011	0.010	0.016	1	1	1	0.99	0.97	0.98	0.96	0.79	0.91	0.89	0.78	0.85
KRAW	10	0.014	0.013	0.015	1	1	1	0.99	0.99	1	1	0.99	1	0.93	0.93	0.96
KRAW	11	0.053	0.036	0.037	0.99	1	1	0.83	0.98	0.93	0.0001	0.9	0.04	0.0001	0.85	0.16
NWSC	10	0.010	0.008	0.010	1	1	1	0.97	0.99	0.99	0.53	0.89	0.99	0.62	0.84	0.93
NWSC	6	0.323	0.443	0.288	1	1	1	1	1	1	1	1	1	1	0.99	1
NWTG	21	0.011	0.010	0.013	1	1	1	0.98	1	0.97	0.82	1	0.63	0.8	0.97	0.69
NWTG	6	0.318	0.427	0.280	1	1	1	1	1	1	1	1	1	1	0.99	1
PROS	11	0.660	0.822	0.596	1	1	1	1	1	1	1	1	1	1	1	1
PROS	ZYWI	0.021	0.011	0.026	1	1	1	0.97	0.95	0.97	0.71	0.0001	0.74	0.73	0.0001	0.75
TRNW	4	0.009	0.008	0.016	1	1	1	1	0.99	0.94	1	0.99	0.23	1	0.93	0.4
TRNW	6	0.279	0.372	0.242	1	1	1	1	1	1	1	1	1	1	0.99	1
TRNW	KRAW	0.208	0.206	0.204	1	1	1	1	1	1	1	1	1	1	1	1
ZYWI	18	0.407	0.552	0.369	1	1	1	1	1	1	1	1	1	0.99	1	1
ZYWI	7	0.033	0.029	0.039	1	0.0001	0.34	0.99	0.0001	0.42	0.98	0.0001	0.0001	0.91	0.0001	0.0001

Table 6: The effect of the damping function on the adjusted coordinates

Pt. No.	*LSQ*				*QDF-trad*				*EDF-trad*				*QDF-smpl*				*EDF-smpl*			
	dX	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*	*dX*	*dY*	*dZ*	*dL*
4	0.001	0.000	– 0.010	0.0060	0.001	– 0.001	– 0.011	0.0063	0.001	– 0.001	– 0.011	0.0062	0.003	0.000	– 0.005	0.0035	0.003	0.000	– 0.006	0.0040
6	– 0.011	– 0.010	– 0.004	0.0088	– 0.011	– 0.011	– 0.004	0.0092	– 0.010	– 0.011	– 0.003	0.0090	– 0.005	– 0.009	0.003	0.0061	– 0.004	– 0.009	0.003	0.0060
7	0.035	0.002	0.019	0.0232	0.024	0.008	0.014	0.0167	0.024	0.008	0.012	0.0165	0.032	0.006	0.010	0.0197	0.032	0.006	0.010	0.0196
9	0.008	0.025	0.028	0.0223	0.007	0.024	0.025	0.0203	0.006	0.024	0.025	0.0201	0.004	0.020	0.025	0.0183	0.004	0.019	0.025	0.0181
10	0.003	– 0.002	0.004	0.0031	0.003	– 0.004	0.004	0.0037	0.003	– 0.004	0.004	0.0033	0.007	– 0.002	0.004	0.0048	0.007	– 0.003	0.004	0.0049
11	– 0.003	0.010	0.012	0.0091	– 0.015	– 0.002	0.006	0.0094	– 0.014	– 0.002	0.006	0.0089	– 0.009	0.004	– 0.003	0.0058	– 0.009	0.003	– 0.003	0.0056
18	– 0.002	0.010	0.007	0.0071	0.000	0.003	– 0.002	0.0022	0.001	0.003	– 0.002	0.0022	0.004	0.004	– 0.001	0.0034	0.005	0.003	– 0.001	0.0034
21	– 0.001	– 0.006	0.006	0.0049	– 0.001	– 0.007	0.004	0.0050	– 0.001	– 0.007	0.004	0.0049	0.003	– 0.007	0.004	0.0049	0.003	– 0.007	0.004	0.0049
Average				0.0106	Average			0.0091	Average			0.0089	Average			0.0083	Average			0.0083
–				–	Percent change			~ 14 %	Percent change			~ 16 %	Percent change			~ 21 %	Percent change			~ 21 %

by the author). Too low value of the parameter (for the simplified method it was established that $k = 1.6$) may result in too dramatic damping of some observations, which in effect weakens the entire geometric system of the network.

6 Summary and conclusions

The aim of the measurements as well as the numerical processing and analysis of the results, was to evaluate the possibility of applying the damping function in reducing the effect of long GPS vector errors on the results of network adjustment. The study examined the possibility of application of the classical, satellite and integrated methods of measurement in determination of absolute dislocations. The comparative analysis could be reduced to specifying the results for different variants of the determined point network, depending on the method of referring to the reference points of the EUPOS system. Two damping functions and two methods of standardisation of adjustment corrections were used at the stage of numerical processing of results of GPS measurements [7]. A method of adjustment of an integrated network is also provided. Calculations and specifications of coordinates for different variants were prepared in the geocentric system of ellipsoid GRS80 (to avoid reduction and projecting corrections and the errors related to them).

The general conclusions drawn from the comparative analyses are as follows:

– long reference vectors (over 10 km) bear a systematic error which results in imprecise establishing the position of points (such errors may be important in analysing absolute dislocations),
– errors of establishing the position (coordinates) are not directly transferred to the calculated distances between points (important in determination of relative dislocations),
– attaching a set of local classical observations (without referring to the reference points) to the process of GPS observation (vector) adjustment improves the precision of establishing the absolute positions,
– owing to application of the damping function (with the appropriate criteria) it is possible to reduce the position determination error by ca. 20 %,
– a better result can be achieved (further reduction of the position determination error) by empirical selection of the appropriate values of the parameters of the damping function (k_0, k); if the value of parameter k is too low, correct observations may be dampened or rejected, which may weaken the network configuration,
– the final effect is more influenced by the method used to standardise the adjustment corrections and selection of the damping function parameters (traditional or simplified method [7]), than the type of the function applied (*EDF* [6] or *QDF* [5]).

Acknowledgements

This study has been financed by funds for scientific research in the years 2007 – 2009 as a research project (Nr. N N526 2094 33).

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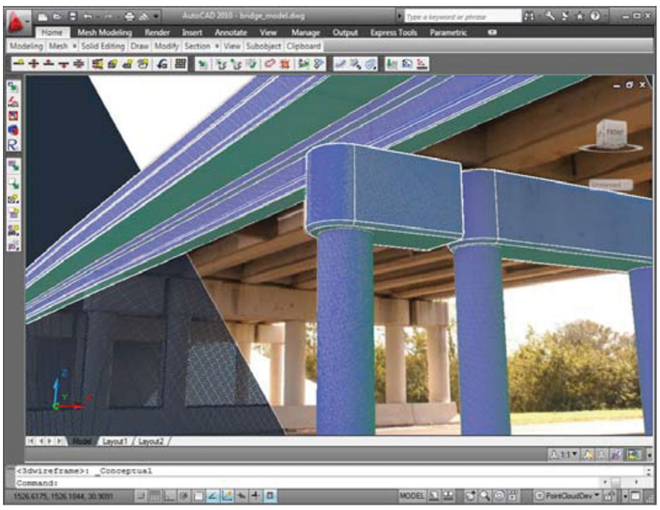
PRODUKTINFORMATION

3D-Punktwolken in AutoCAD auswerten



Messungen mit 3D-Laserscannern ergeben sehr schnell Millionen einzelner Punkte, jedoch ohne Informationen, wie sich die Punkte zueinander verhalten und was sie eigentlich abbilden. PointCloud und PointCloud Pro von kubit ermöglichen die Darstellung und Auswertung von Punktwolken unmittelbar in AutoCAD. Jeder Punkt einer 3D-Datenwolke kann mit den AutoCAD eigenen Befehlen genutzt werden (OFANG). Darüber hinaus bietet PointCloud zahlreiche eigene Befehle zur effizienten Analyse der 3D-Laserscanner-Daten. Koordinatensysteme, Polylinien, Ebenen und Zylinder können an Punktwolkenbereiche angepasst werden. Das Verschneiden zweier Ebenen liefert schnell Haus- oder Bordsteinkanten, mit drei Ebenen lassen sich Ecken präzise bestimmen. Die Ergebnisse dieser Auswertungen sind Standard-2D-Pläne oder 3D-Modelle.

Das neue PointCloud Pro 5 erweitert AutoCAD um ein photogrammetrisches Mehrbild-Analyseverfahren.

Auch ohne Punktwolken lassen sich so aus digitalen Fotos dreidimensionale Objekte konstruieren. Orientierte Fotos werden in PointCloud direkt erzeugt und „abgepaust“. Die auf dem zweidimensionalen Foto gezeichneten Linien erscheinen zeitgleich in der 3D-Punktwolke an geometrisch korrekter Stelle.

Lizenzinhaber von *PointCloud* können Punktwolken zusammen mit einer kostenlosen, voll funktionsfähigen FreeEdition von *PointCloud* weiter geben. Damit wird es Dienstleistern ermöglicht, gemessene Punktwolken an Kunden zur weiteren Bearbeitung in AutoCAD weiter zu geben, ohne dass die Kunden eine *PointCloud* Lizenz benötigen.

Die aktuellen Versionen 5 von PointCloud und PointCloud Pro sind kompatibel zu AutoCAD 2007 bis AutoCAD 2010 und allen auf diesen aufsetzenden Programmen. Sie unterstützen 64-bit Betriebssysteme. Für Interessenten besteht die Möglichkeit, die Software kostenfrei zu testen.

Mit PointCloud Pro 5 können neben Punktwolken auch Fotos zur Auswertung in AutoCAD importiert und orientiert werden (siehe Abbildung).

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