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History of Considering Refraction in Leveling

Seit Jahrzehnten ist der Einfluss der Refraktion auf die Ergebnisse von trigonometrischen und geometrischen Präzisionsnivellements untersucht worden, um Korrektionen zu bestimmen, die auf mehr oder weniger aufwendige Weise ermittelt werden müssen. Die historische Entwicklung und die Vorteile der Benutzung der Ähnlichkeitstheorie werden aufgezeigt.

1 Early Research

In 1669 the French astronomer Pikar (1620–1682) has calculated the ratio of the radius R_3 of the Earth to the radius of the curvature of the optical beam R_0 to 0.18. Subsequently the ratio $k = R_3/R_0$ has received the name of a coefficient of terrestrial refraction and was widely used in geodesy. Knowing k, it is possible to find the correction in the height h measured by one sided trigonometric leveling

$$\rho = 0.55 S^2 R_3^{-1} k, \tag{1}$$

where S is the distance between points. Having measured $\rho_0 = h - h_0$, it is possible to find the coefficient of refraction:

$$k = 2\rho_0 S^{-2} R_3. \tag{2}$$

Thus h_0 can be determined from results of two sided trigonometric or geometrical leveling.
Measurements of the coefficient k with the purpose of accepting its most probable value were made in a number of countries during the 18th and 19th centuries. For practical calculations [8] the value of $k = 0.13$ was determined by Gauss.
Extensive research by round-the-clock measurements of the coefficient of refraction on lines of different length was executed in the USSR [7]. They have shown, that any measurements of heights by trigonometric leveling are accompanied by regular errors due to the influence of refraction. It is impossible to choose periods of time during the day when the influence of refraction could be neglected.

2 Jordan's Formula and Monin-Obuchov's Theory of Similarity

Jordan's formula [8] developed by him in 1876 for the definition of a „local" coefficient of refraction in a separate point of a beam at the height z with fixed values for the meteorological elements (atmospheric pressure p (mb), temperatures of air T (κ) and its gradient γ_Z ($\kappa/_M$)) lays the basis for a «meteorological direction»:

$$\kappa = 503 p T^{-2} (0.0342 - \gamma_Z) \tag{3}$$

The mark of the gradient $\gamma_Z = -dT/dz$ is considered positive when the temperature of air drops with height.
The change of temperature of the air with height in a ground layer (in conditions of indifferent thermal stratification) is described by the known formula given in [1]:

$$\gamma_Z = \frac{T_1 - T_2}{z \cdot In \frac{z_2}{z_1}} = \frac{T_*}{z} \tag{4}$$

Here are T_1 and T_2 temperatures of air at heights z_1 and z_2, respectively. T_* – is a temperature scale (dynamic temperature) in the Monin-Obuhov theory of similarity. Equation (4) is established theoretically and confirmed in numerous experiments. It expresses the logarithmic law of temperature change of air with height in the ground layer of the atmosphere.
A formula close to equation (4) was used after substitution in equation (3) by the authors [7] for calculation of a „local" coefficient κ, determined by meteorological facts. The statistical processing of the numbers k and κ, found from simultaneous geodetic and meteorological measurements, has been specified by the absence of a practical opportunity to calculate corrections by direct meteorological measurements.
After investigating the influence of refraction in a precision leveling Entin [24] has come to a similar conclusion. He subjected the formulas already known for calculation of corrections to an experimental check and came to the conclusion that the existing ways of measuring vertical gradients do not give the characteristics of a temperature field for determining a trajectory of distribution of a light beam.
In the next years such research proceeded. The results have expanded a circle of representations about a physical picture of the phenomenon [1, 22, 25].

3 Temperature Gradient and Turbulence

In the years1960–70 the basic interest of researchers was directed to the realization of Näbauer's idea about a determination of refraction using dispersive angles caused by the difference of refraction of light rays with various wave lengths. This has been put forward in 1924 and 1929, respectively. Laboratory models for such a device were developed in a number of laboratories [25]. But these devices have not found practical applications. One of the difficulties, facing this way, is connected to the influence of atmospheric turbulence, which causes casual fluctuations of the dispersive angle. The accuracy of measurements should not be lower than 0.002.

The following stage of research is based on an idea of Brunner for the determination of a temperature gradient by measurements of the parameters of turbulence C_n^2, l_0 (the structural characteristic of fluctuations for the parameter of refraction and the internal scale of turbulence). This is realized in the works [2 ,4, 5, 26], where the description of the measuring equipment allowing to find specified characteristics using modern technologies is given. The accuracy for calculating the gradient is limited to the third figure after the point. In the monographs [22, 25] the intensity of turbulence is estimated inside a pipe. The question about a practical opportunity using this method remains open.

4 The Integrated Temperature Gradient and the Theory of Similarity

After substitution of equation (3) in (1) we get:

$$\rho = 39.5 \cdot \frac{p}{T^2} \cdot S^2 \cdot (0.0342 - \gamma_z) \cdot 10^{-6}. \qquad (5)$$

Having measured the size of the parameter $\rho_0 = h - h_0$, we receive the value of an integrated temperature gradient

$$\bar{\gamma}_Z = 0.0342 - \left(\frac{\rho_0 \cdot T^2}{39.5pS^2}\right) \cdot 10^6. \qquad (6)$$

According to research given in [1] the accuracy of measurements of an integrated gradient is estimated to 1–5 10^{-4} $\kappa/_{M}$. This cannot be achieved by any other means at the present stage. Such an accuracy for $\bar{\gamma}_Z$ is necessary for the calculation of corrections with an accuracy sufficient for practical purposes.

Comparing the gradients $\bar{\gamma}_Z$ and γ_Z, determined by geodetic and meteorological means it is necessary to pay attention to their similar and distinctive attributes [17]. The parameters $\bar{\gamma}_Z$ and γ_Z have an identical dimension, an identical physical nature and are developed under identical physical conditions. At the same time they differ both concerning accuracy and the object of measurements. If the gradient $\bar{\gamma}_Z$ represents the integrated characteristic of the temperature drop of an optical beam along its way, the gradient γ_Z characterizes the change of the temperature only in the vertical. The error of measurement of this gradient does not fall outside the limits of the second or third figure after the point. This is the reason for the low efficiency of the „meteorological“ way.

It is necessary to notice that the coefficient of refraction in equation (2) contains only one variable $\bar{\gamma}_Z$. The constant R_3 does not influence the physical process and does not play the essential role in the calculation of corrections. Therefore at the present stage the concept about the coefficient of refraction has lost practical interest.

The solution of a problem [10, 14, 19] is based on the use of the determined $\bar{\gamma}_Z$ from determining γ_Z, z_{av} (parameters received by measurements on a skilled object). The parameter γ_1 represents a gradient of temperature of air at the height z = 1 m. The known technique [9] allows finding the gradient γ_1 only on basis of psychrometer measurements. It extremely simplifies the process of its determination under field conditions. For determination of the geometrical parameter z_{av} it is possible to use topographical maps.

The parameters γ_1 and z_{av} characterize conditions under which geodetic measurements were made. The sense of their use consists in choosing a concrete value from the big variety of gradients $\bar{\gamma}_Z$ arising on the object during the realization of geodetic works. Having a rather limited volume of measurements for $\bar{\gamma}_Z$, γ_1, z_{av} they are processed dimensionless (numbers of similarity). This allows to establish a function of transition for $\bar{\gamma}_Z = f(\gamma_1,\ z_{av})$, measured on the model under natural conditions, irrespective of place and time, realized by an experiment.

The theory of similarity has wide applications in science and technique [23]. In particular it concerns such branches of knowledge which are connected to studying the movement of a liquid or a gas: hydromechanics, physics, meteorology (physics of a ground layer) etc. Successful applications of the method of similarity in new areas of engineering always represent a new technical decision. Thus, it is necessary to mention, that on this account the theory of similarity does not require any strict recommendations. It is important to have a deep understanding of the physical picture of the phenomenon. A first step for applying the method is a selection of parameters determining the physical process. Thus, it is important to exclude minor factors.

As an analogy (prototype) of the offered way on the account of refraction it is necessary to consider a known way for the definition of corrections with the help of a factor *k*. The lack of the known way are the restrictions arising from the factor *k* determined at one object and using it for other objects where such measurements are not generally made. It is caused by the distinction of the parameters T_* and z in formula (4), and consequently for γ_Z in different objects. A similar picture takes place in the consideration of the parameters $\bar{\gamma}_Z$, γ_1, z_{av}.

The essence of the offered way consists in the following. We consider two physical phenomena described by identical formed equations (4):

$$\gamma'_Z = \frac{T'_*}{z'};\ \gamma''_Z = \frac{T''_*}{z''};$$

The first of them we name modeling and the second one natural. We divide them and solve the obtained parity relative to γ''_Z:

$$\gamma''_Z = \gamma'_Z\left(\frac{T''_*}{T'_*}\right) \cdot \left(\frac{z'}{z''}\right).$$

Let's replace the parameters γ_Z, T_*, z by the known (measured) quantities $\bar{\gamma}_Z$, γ_1, z_{av}:

$$\bar{\gamma}''_Z = \bar{\gamma}'_Z\left(\frac{\gamma''_1}{\gamma'_1}\right)^n \cdot \left(\frac{z'_{av}}{z''_{av}}\right)^m \qquad (7)$$

The exponents *m*, *n* represent constant numbers (empirical constants) and their definition is considered in [14, 19, 21]. They characterize a degree of influence on the parameters

γ_1, z_{av} and the required variable $\bar{\gamma}_Z$. We group these quantities according to their indices, worth on the right to an equal-sign in equation (7):

$$\bar{\gamma}_Z'' = \gamma_Z' \frac{(z_{av}')^m}{(\gamma_1')^n} \cdot \frac{(\gamma_1'')^n}{(z_{av}'')^m}. \qquad (8)$$

Having rejected the indices, we can write:

$$\bar{\gamma}_Z = a \cdot \frac{\gamma_1^n}{z_{av}^m}, \qquad (9)$$

where the constant a is made of the quantities measured on an experimental object. The values of the factors $n = 0.2$; $m = 1$ were obtained by processing the experimental data using the method of similarity that allows to find the factor a on the basis of individual measurements of the quantities $\bar{\gamma}_Z$, γ_1, z_{av} on a model object. The volume of the measurements differs a little from the known definitions of the coefficient k.

Let's write equation (8) as:

$$\bar{\gamma}_Z'' = \bar{\gamma}_Z' \cdot K,$$

where K is a transitive multiplier, allowing to transform the gradient measured on the model to natural conditions. As we see, the accuracy of defining this factor K depends on the accuracy of measurements of an integrated gradient on an experimental object. A necessary condition for such recalculation is the similarity in the distribution of the temperature of air with height (thermal stratification) on the experimental and natural sites. The logarithmic law (4) which is well traced in conditions of unstable stratification (in a warm season in the afternoon above land) serves as the foundation for the existence of similarity.

Thus, the task is reduced to the measurement of two quantities γ_1 and z_{av} on a natural object. Their substitution in equation (9) and then in formula (5) allows to calculate the correction for an excess measured by an unilateral leveling. The procedure and the results of the experimental check of the method are considered in [3, 6, 20].

Refraction at a two sided leveling is considered in two variants: with the use of one sided mutual return measurements [16, 21] and by the method of an auxiliary basis [16, 19]. The application of the last demands some additional geodetic measurements.

The way of determining corrections in a precision leveling is considered in [15, 18, 19,].

So, the considered way based on material of numerical experiments with subsequent processing of the results by the method of similarity solves the problem on the account of refraction in precision and trigonometric leveling by rather simple means.

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Abstract

Since many decades the regular influence of refraction on the results of trigonometric and precision geometric leveling is considered as one of the major geodetic problems. Research has been executed to find corrections:

- **using a coefficient for terrestrial refraction,**
- **by meteorological measurements in order to determine a temperature gradient,**
- **by measuring a dispersive angle,**
- **by determining a temperature gradient based on measurements of characteristic parameters of the atmosphere's turbulence,**
- **on the basis of processing material of experiments by a method of similarity.**