A New Solution Model of Nonlinear Integral Adjustment Including Different Kinds of Observing Data with Different Precision

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Der Beitrag zeigt einen Weg, nichtlineare Beobachtungen unterschiedlicher Art in einer gemeinsamen Ausgleichung zu behandeln.

In the data process of the construction of digital city and digital nation, and the modern deformation monitoring, many kinds of measurements with different precision containing geometric and physical data are captured. The distribution types of these data can be divided into normal distribution, Laplace's distribution, normal mixed distribution and so on. But the main distribution is normal. Meantime the relations between these observing data and the unknown parameters are nonlinear in most of the cases. So far different kinds of observing data with different precision usually can independently be processed with the method of classical linear least square, respectively. Obviously it is not scientific and accurate. Now the theory of data process with nonlinear least square method has been an important object to be studied in the field of surveying and mapping all over the world. So the International Association of Geodesy (IAG) thinks it as an important question for study. Now it is beginning to study the data process with the nonlinear least square method, supposing that the nonlinear functions are continuous and derivative. Therefore we can get the derivatives of the target function and calculate a group of best parameters to make the nonlinear target function to be extreme value. The method is complex and takes more calculating load. The nonlinear integral least square adjustment containing different kinds of observing data with different precision is the same as the above. To solve the nonlinear integral least square adjustment including different kind of observing data with different precision, a new solution model and its calculating method are put forward. The new method doesn't derive the derivative of the relative functions. The calculating load of the new nonlinear model is less than the existing methods and easy to be calculated.

1 Solving model of nonlinear integral least square adjustment with non-derivative of the error functions

To study the problem conveniently, we suppose that there are two kinds of observing data with different precision. Carrying out the integral joined adjustment, the observing data can be divided into several groups according to the kind and precision of data and the nonlinear error function of each group should be founded. Now suppose there are two groups of observing data, $\underset{m\times 1}{L_1}$ and $\underset{n\times 1}{L_2}$, whose weight matrices are $\underset{m\times m}{P_1}$ and $\underset{n\times n}{P_2}$, and corresponding correct matrices are V_1 and V_2 respectively. In the mean time, $\underset{m\times 1}{L_1}$ is not relative to $\underset{n\times 1}{L_2} \cdot \underset{s\times 1}{d}$ is the unknown parameter matrix. So we can get the nonlinear error function as

$$V_1 = f_1(d_1 d_2 \ldots d_s) - L_1 \quad P_1 \quad \text{who belong to normal distribution} \tag{1}$$

$$V_2 = f_2(d_1 d_2 \ldots d_s) - L_2 \quad P_2 \quad \text{who belong to Lalace's distribution}$$

In order to obtain the more precise adjusting results, it is important to understand the weights of all kinds of observing data when carrying out the integral process of different kinds of data with different precision. One method to solve the problem is often used to get the appropriate weight ratio of all kinds of data. Supposing the weight of normal data is P_1 and that of Laplace's data is $P_2 = P_1 C/|V_2|$, in which C can be obtained from the function table of standard normal accumulated distribution. In general, $|V_2| > C$. To get further the more reliable weight ratio, then the more reliable variance of all kinds of observing data can be calculated with the posteriori method. So we get the more appropriate weight ratio. From equation (1), we can get the nonlinear integral least square adjustment model as

$$\min F(d) = V_1^T P_1 V_1 + V_2^T P_2 V_2 = \sum_{i=1}^{m} P_{1i} V_{1i}^2 + \sum_{j=1}^{n} P_{2j} V_{2j}^2 = \sum_{i=1}^{m} P_{1i}(f_{1i}(d) - l_{1i})^2 + \sum_{j=1}^{n} P_{2j}(f_{2j}(d) - l_{2j})^2$$

$$= \sum_{i=1}^{m} P_{1i}\varphi_{1i}^2(d) + \sum_{j=1}^{n} P_{2j}\varphi_{2j}^2(d) = F_1(d) + F_2(d) \tag{2}$$

whose target function is the square sum of $F(d) = F_1(d) + F_2(d)$. There have been some methods to solve the model (2). In the calculating process, we must get the first power and second power derivatives of the target function, and S-dimensional power function group must be solved repeatedly to get the search direction. It is complex to carry on the work and takes more time. Therefore we put forward a non-derivative method to solve the problem in the paper. We all know that the stable solution of the model (2) can be obtained by the essential condition of the extreme value. According to the essential condition of its extreme value, model (2) must satisfy $\nabla F(d) = 0$, that is

$$\nabla F = \begin{pmatrix} \frac{\partial\varphi_{11}(d)}{\partial d_1} & \frac{\partial\varphi_{12}(d)}{\partial d_1} & \cdots & \frac{\partial\varphi_{1m}(d)}{\partial d_1} & \frac{\partial\varphi_{21}(d)}{\partial d_1} & \frac{\partial\varphi_{22}(d)}{\partial d_1} & \cdots & \frac{\partial\varphi_{2n}(d)}{\partial d_1} \\ \frac{\partial\varphi_{11}(d)}{\partial d_2} & \frac{\partial\varphi_{12}(d)}{\partial d_2} & \cdots & \frac{\partial\varphi_{1m}(d)}{\partial d_2} & \frac{\partial\varphi_{21}(d)}{\partial d_2} & \frac{\partial\varphi_{22}(d)}{\partial d_2} & \cdots & \frac{\partial\varphi_{2n}(d)}{\partial d_2} \\ \vdots \\ \frac{\partial\varphi_{11}(d)}{\partial d_s} & \frac{\partial\varphi_{12}(d)}{\partial d_s} & \cdots & \frac{\partial\varphi_{1m}(d)}{\partial d_s} & \frac{\partial\varphi_{21}(d)}{\partial d_s} & \frac{\partial\varphi_{22}(d)}{\partial d_s} & \cdots & \frac{\partial\varphi_{2n}(d)}{\partial d_s} \end{pmatrix} P\varphi(d)$$

$$P = \begin{pmatrix} P_{11} \\ & P_{12} \\ & & \ddots \\ & & & P_{1m} \\ & & & & P_{21} \\ & & & & & P_{22} \\ & & & & & & \ddots \\ & & & & & & & P_{2n} \end{pmatrix}, \quad \varphi(d) = \begin{pmatrix} \varphi_{11}(d) \\ \varphi_{12}(d) \\ \vdots \\ \varphi_{1m}(d) \\ \varphi_{21}(d) \\ \varphi_{22}(d) \\ \vdots \\ \varphi_{2n}(d) \end{pmatrix}$$

Let $J^T = \begin{pmatrix} \frac{\partial\varphi_{11}(d)}{\partial d_1} & \frac{\partial\varphi_{12}(d)}{\partial d_1} & \cdots & \frac{\partial\varphi_{1m}(d)}{\partial d_1} & \frac{\partial\varphi_{21}(d)}{\partial d_1} & \frac{\partial\varphi_{22}(d)}{\partial d_1} & \cdots & \frac{\partial\varphi_{2n}(d)}{\partial d_1} \\ \frac{\partial\varphi_{11}(d)}{\partial d_2} & \frac{\partial\varphi_{12}(d)}{\partial d_2} & \cdots & \frac{\partial\varphi_{1m}(d)}{\partial d_2} & \frac{\partial\varphi_{21}(d)}{\partial d_2} & \frac{\partial\varphi_{22}(d)}{\partial d_2} & \cdots & \frac{\partial\varphi_{2n}(d)}{\partial d_2} \\ \vdots \\ \frac{\partial\varphi_{11}(d)}{\partial d_s} & \frac{\partial\varphi_{12}(d)}{\partial d_s} & \cdots & \frac{\partial\varphi_{1m}(d)}{\partial d_s} & \frac{\partial\varphi_{21}(d)}{\partial d_s} & \frac{\partial\varphi_{22}(d)}{\partial d_s} & \cdots & \frac{\partial\varphi_{2n}(d)}{\partial d_s} \end{pmatrix}$, then

$\nabla F(d) = 2J^T P\varphi(d) = 0$. In the vicinity of the initial point $d^{(0)}$, now we discuss a linear function close to the vector function $\varphi(d)$. So

$$\varphi_{11}(d) = \varphi_{11}(d_1^{(0)} d_2^{(0)} \ldots d_s^{(0)}) + \frac{\partial\varphi_{11}(d)}{\partial d_1}(d_1 - d_1^{(0)}) + \frac{\partial\varphi_{11}(d)}{\partial d_2}(d_2 - d_2^{(0)}) + \ldots\ldots + \frac{\partial\varphi_{11}(d)}{\partial d_s}(d_s - d_s^{(0)})$$

$$= \varphi_{11}(d_1^{(0)} d_2^{(0)} \ldots d_s^{(0)}) - \frac{\partial\varphi_{11}(d)}{\partial d_1} d_1^{(0)} - \frac{\partial\varphi_{11}(d)}{\partial d_2} d_2^{(0)} \ldots + \frac{\partial\varphi_{11}(d)}{\partial d_s} d_s^{(0)} + \frac{\partial\varphi_{11}(d)}{\partial d_1} d_1 + \frac{\partial\varphi_{11}(d)}{\partial d_2} d_2$$

$$+\ldots\ldots \frac{\partial\varphi_{11}(d)}{\partial d_s} d_s = C_{11} + a_{11}d_1 + a_{12}d_2 + \ldots + a_{1s}d_s$$

whose approximate function is

$$l_{11}(d) = C_{11} + a_{11}d_1 + a_{12}d_2 + \ldots + a_{1s}d_s$$

With the same method as the above, we can get

$$\begin{pmatrix} l_{12}(d) = C_{12} + a_{21}d_1 + a_{22}d_2 + \ldots + a_{2s}d_s \\ \vdots \\ l_{1m}(d) = C_{1m} + a_{m1}d_1 + a_{m2}d_2 + \ldots + a_{ms}d_s \\ l_{21}(d) = C_{21} + b_{11}d_1 + b_{12}d_2 + \ldots + b_{1s}d_s \\ \vdots \\ l_{2n}(d) = C_{2n}\phi + b_{n1}d_1 + b_{n2}d_2 + \ldots + b_{ns}d_s \end{pmatrix} \tag{3}$$

Then

$$l(d) = Ad + C \tag{4}$$

Calculating the derivative of equation (4), we can get

$$\nabla l(d) = A \tag{5}$$

To make $\tilde{F}(d) = l(d)^T\ Pl(d)$ take the place of $F(d) = \varphi(d)^T\ P\varphi(d)$ in the vicinity of $d^{(k)}$, we can obtain

$$\nabla\tilde{F}(d) = 2\nabla l(d)^T\ Pl(d) = 0 \tag{6}$$

From equations (4), (5) and (6), we can get $\nabla\tilde{F}(d) = 2A^T\ P(Ad + C) = 0$. Now let $C_K = l(d^{(k)})\ - A_k d^{(k)} = \varphi(d^{(k)}) - A_k d^{(k)}$ and $d = d^{(R+1)}$, then

$$d^{(k+1)} = d^{(k)} - (A_k^T PA_k)^{-1} A_k^T P\varphi(d^{(k)}) \tag{7}$$

in which the matrix A_k must satisfy

$$\Delta\varphi_k = A_k \Delta d_k \tag{8}$$

where $\Delta d_k = [d_1* - d_1^{(k)},\ d_2* - d_2^{(k)}, \ldots, d_s* - d_s^{(k)}]$, $\Delta\varphi_k = [\Delta\varphi_{11}^{(k)}, \Delta\varphi_{12}^{(k)}, \ldots, \Delta\varphi_{1m}^{(k)}, \Delta\varphi_{21}^{(k)}, \Delta\varphi_{22}^{(k)}, \ldots, \Delta\varphi_{2n}^{(k)}]$, $\Delta\varphi_{11}^{(k)} = \varphi_{11}(d*) - \varphi_{11}(d^{(k)})$, $\Delta\varphi_{12}^{(k)} = \varphi_{12}(d*) - \varphi_{12}(d^{(k)})$, $\Delta\varphi_{1m}^{(k)} = \varphi_{1m}(d*) - \varphi_{1m}(d^{(k)})$, $\Delta\varphi_{21}^{(k)} = \varphi_{21}(d*) - \varphi_{21}(d^{(k)})$, $\Delta\varphi_{22}^{(k)} = \varphi_{22}(d*) - \varphi_{22}(d^{(k)})$, and $\Delta\varphi_{2n}^{(k)} = \varphi_{2n}(d*) - \varphi_{2n}(d^{(k)})$. Δd_k is a $s \times s$ matrix, $\Delta\varphi_k$ is a $(m+n) \times s$ matrix, $d*$ is the current approximate value of the most optimal point, and $d^{(k)}$ is the initial value pre-given. In general, the rank of Δd is full. Therefore equation (8) can only determine a $(m+n) \times s$ matrix, that is

$$A_k = \Delta\varphi_k(\Delta d_k)^{-1} \tag{9}$$

Let

$$C_k = \varphi(d^{(k)}) - \Delta\varphi_k(\Delta d_k)^{-1} d_\phi^{(k)} \tag{10}$$

then the approximate linear function of $\varphi(d)$ in the vicinity of $d^{(k)}$ can be expressed as $l_k(d) = \Delta\varphi_k(\Delta d_k) - (d* - d^{(k)}) + \varphi(d^{(k)})$. So from equation (9), we can get

$$A_k^T PA_k = [\Delta d_k)^{-1}]^T \Delta\varphi_k^T P\Delta\varphi_k(\Delta d)^{-1} \tag{11}$$

As the rank of $\Delta\varphi_k$ is full, $A_k^T A_k$ is a positive definite matrix. From equation (11), we can obtain

$$(A_k^T PA_k)^{-1} = \Delta d_k(\Delta\varphi_k^T P\Delta\varphi_k)^{-1}\Delta d_k^T \tag{12}$$

With the method of general nonlinear least square, from equation (7) we can get

$$d^{(k+1)} = d^{(k)} - \Delta d_k(\Delta\varphi_k^T P\Delta\varphi_k)^{-1}\Delta d_k^T[(\Delta d_k)^T]^{-1}\Delta\varphi_k^T P\varphi(d^{(k)}) \tag{13}$$

Let $d_r^{(k)} = d^{(k)}$ and $r = 1, 2 \ldots,\ s$, then $d_r* = d_r^{(k)} + (d_r^{(k-1)} - d_r^{(k)})e_r$ in which $e_r^T = (0\ 0 \ldots \overset{r}{1} 0 \ldots 0)$. If $h_r^{(k)} = (d_r^{(k-1)} - d_r^{(k)} \neq 0$, Δd_k is a diagonal matrix, that is

$$\Delta d_k = \begin{bmatrix} h_1^{(k)} & & & 0 \\ & h_2^{(k)} & & \\ & & \ddots & \\ 0 & & & h_s^{(k)} \end{bmatrix}$$

And the matrix $\Delta\varphi_k$ can be expressed as the following

$$\Delta\varphi_k = \left[\Delta\varphi_{11}^{(k)}\Delta\varphi_{12}^{(k)}\ldots\Delta\varphi_{1m}^{(k)}\Delta\varphi_{21}^{(k)}\Delta\varphi_{22}^{(k)}\ldots\Delta\varphi_{2n}^{(k)}\right]^T$$

$$\begin{aligned}\Delta\varphi_{11}^{(k)} = {} & [\varphi_{11}(d_1^{(k)} + h_1^{(k)}\ d_2^{(k)}\ldots d_s^{(k)}) - \varphi_{11}(d_1^{(k)}d_2^{(k)}\ldots d_s^{(k)}),\ \varphi_{11}(d_1^{(k)}d_2^{(k)} + h_2^{(k)}d_3^{(k)}\ldots d_s^{(k)}) \\ & -\varphi_{11}(d_1^{(k)}d_2^{(k)}\ldots d_s^{(k)}),\ldots,\varphi_{11}(d_1^{(k)}d_2^{(k)}\ldots d_s^{(k)} + h_s^{(k)}) - \varphi_{11}(d_1^{(k)}d_2^{(k)}\ldots d_s^{(k)})] = [\varphi_{11}(d^{(k)} + h_1^{(k)}e_1) \\ & -\varphi_{11}(d^{(k)}),\ \varphi_{11}(d^{(k)} + h_2^{(k)}e_2) - \varphi_{11}(d^{(k)}),\ldots,\varphi_{11}(d^{(k)} + h_s^{(k)}e_s) - \varphi_{11}(d^{(k)})]\end{aligned}$$

$$\Delta\varphi_{12}^{(k)} = [\varphi_{12}(d^{(k)} + h_1^{(k)}e_1) - \varphi_{12}(d^{(k)}),\ \varphi_{12}(d^{(k)} + h_2^{(k)}e_2) - \varphi_{12}(d^{(k)}),\ldots\varphi_{12}(d^{(k)} + h_s e_s) - \varphi_{12}(d^{(k)})]$$

$$\vdots$$

$$\Delta\varphi_{1m}^{(k)} = [\varphi_{1m}(d^{(k)} + h_1^{(k)}e_1) - \varphi_{1m}(d^{(k)}),\ \varphi_{1m}(d^{(k)} + h_2^{(k)}e_2) - \varphi_{1m}(d^{(k)}),\ldots,\varphi_{1m}(d^{(k)} + h_s^{(k)}e_s) - \varphi_{1m}(d^{(k)})]$$

$$\Delta\varphi_{21}^{(k)} = [\varphi_{21}(d^{(k)} + h_1^{(k)}e_1) - \varphi_{21}(d^{(k)}),\ \varphi_{21}(d^{(k)} + h_2^{(k)}e_2) - \varphi_{21}(d^{(k)}),\ldots,\varphi_{21}(d^{(k)} + h_s^{(k)}e_s) - \varphi_{21}(d^{(k)})]$$

$$\Delta\varphi_{2n}^{(k)} = [\varphi_{2n}(d^{(k)} + h_1^{(k)}e_1) - \varphi_{2n}(d^{(k)}),\ \varphi_{2n}(d^{(k)} + h_2^{(k)}e_2) - \varphi_{2n}(d^{(k)})\ldots\varphi_{2n}(d^{(k)} + h_s^{(k)}e_s) - \varphi_{2n}(d^{(k)})]$$

$\Delta\varphi_k$ can again be expressed as

$$\Delta\varphi_k = [\varphi_1(d^{(k)} + h_1^{(k)}e_1) - \varphi_1(d^{(k)}),\ \varphi_2(d^{(k)} + h_2^{(k)}e_2) - \varphi_2(d^{(k)}),\ldots,\varphi_s(d^{(k)} + h_s^{(k)}e_s) - \varphi_s(d^{(k)})] = \Delta\varphi(d^{(k)}h^{(k)}) \tag{13}$$

So

$$A_k = \Delta\varphi_k(\Delta d_k)^{-1} = \left[\frac{1}{h_1^{(k)}}(\varphi_1(d^{(k)} + h_1^{(k)}e_1) - \varphi_1(d^{(k)})\right],$$

$$\frac{1}{h_2^{(k)}}[\varphi_2(d^{(k)} + h_2^{(k)}e_2) - \varphi_2(d^{(k)})],$$

$$\ldots,\frac{1}{h_s^{(k)}}\left[(\varphi_s(d^{(k)} + h_s^{(k)}e_s) - \varphi_s(d^{(k)})\right]$$

From the comparison with equation (4), we can know A_k is a matrix composed of the difference of function $\varphi(d)$, which takes the place of the first power derivative matrix of $\varphi(d)$ in equation (4). When $h_1^{(k)} = h_2^{(k)} = \ldots = h_s^{(k)} = h^{(k)}$, then we can get

$$A_k = \frac{1}{h^{(k)}}[\varphi_1(d^{(k)} + h_1^{(k)}e_1) - \varphi_1(d^{(k)}),\ \varphi_2(d^{(k)} + h_2^{(k)}e_2) - \varphi_2(d^{(k)}),\ \ldots,\varphi_s(d^{(k)} + h_s^{(k)}e_s) - \varphi_s(d^{(k)})] = \frac{1}{h^{(k)}}\Delta\varphi(d^{(k)}h^{(k)}) \tag{15}$$

To substitute equation (15) into equation (11) and (7), we can get the calculating model

$$d^{(k+1)} = d^{(k)} - h^{(k)}[\Delta\varphi(d^{(k)}h^{(k)})^T P\Delta\varphi(d^{(k)}h^{(k)})]^{-1}\cdot\Delta\varphi(d^{(k)}h^{(k)})^T P\varphi(d^{(K)}) \tag{16}$$

which includes the difference instead of the derivative.

2 Non-derivative calculating processes of the nonlinear integral adjustment containing different kinds of observing data with different precision

Step 1: Let the iterative number $k = 0$, the initial weight $P_1^0 = P_2^0 = 1$ and the allowable error $\varepsilon(\varepsilon < 0)$ and give the initial approximate value $d^{(0)}$.

Step 2: Calculate $\varphi(d^{(k)}) = [\varphi_{11}(d^{(k)}),\ \varphi_{12}(d^{(k)}), \ldots, \varphi_{1m}(d^{(k)}),\ \varphi_{12}(d^{(k)}), \ldots, \varphi_{2n}(d^{(k)})]^T$.

Step 3: Solve
$h^{(k)} = 0(\|\varphi(d^{(k)})\|) = \beta(\|\varphi(d^{(k)})\|)\beta \in (0, 1)$.

Step 4: Calculate $\Delta\varphi(d^{(k)}h^{(k)})$.

Step 5: Calculate $d^{(k+1)}$.

Step 6: If $\|d^{(k+1)} - d^{(k)}\| < \varepsilon$, $d^{(k+1)}$ is the optimal solution that satisfies the accuracy claim. Otherwise $d^{(k+1)}$ replaces $d^{(k)}$ and let again $k = k + 1$, then calculate again the weights of two groups of observing data. First we should calculate the square sum of residual errors of data, then solve the variance estimate of the first group of data $\sigma_1 = \frac{V_1^T P_1 V_1}{m-s}$. According to the variance estimate, we can determine the weights of all kinds of observing data, $P_1 = P_1^0 \sigma_0 / \sigma_1^2$ and $P_2 = P_1 C / |V_2|$, in which $\sigma_0 = 1$. Then go to Step 2. Repeat the above processes until the claim is satisfactory.

A new solution method to solve the nonlinear integral least square adjustment containing different kinds of observing data with different precision is put forward in the paper. It is an efficient and simple method. We mainly calculate the function $d^{(k)}$ instead of the derivative in each iterative calculation. So the calculating load is less. In the meantime the method is strict in theory. It has a theoretical and practical significance and opens up a new way to solve the nonlinear integral least square adjustment including different kinds of data with different precision.

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Abstract

In the construction of digital nation and the modern deformation monitoring, different kinds of observing values with different precision are often collected and have the nonlinear relationship with each other. A new solution model of nonlinear dynamic integral least square adjustment including different kinds of observing data with different precision is put forward in the paper, which is not dependent on their derivatives. It is a new method to solve the nonlinear integral adjustment, which has more scientific and practical significance.

Keywords

different kinds of observing data with different precision, nonlinear integral adjustment, non-derivative solving method