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A Fitting Method of Pseudo-polynomial For Solving Nonlinear Parametrical Adjustment*

This paper discusses the condition that the sum of square errors must minimize and its geometrical characteristics. It uses a super curved surface that is a transformed surface. The construction of the surface and its features are discussed in detail.

1 The optimal condition and its geometrical characters

The adjustment model with n observation and m ($m < n$) parameters may be written as

$$\begin{cases} l^r = y^r(x^a) + e^r \\ e^i \sim N(0, g^{ii}) \end{cases} \tag{1}$$

where l^r ($r = 1,2,...n$) represents components of observations; e^r ($r = 1,2,...n$) represents components of error; $y^r(u^a)$ ($a = 1,2,...m$) is assumed to be in a nonlinear map from unknown parametrical sets $\{x^a\}$ to components of adjusted values; e^i is subjected to be a normal distribution and $g^{ij} = E(l^i \cdot l^j) = E(e^i \cdot e^j)$ is the variance-covariance matrices of error vector.

With the method of nonlinear least-squares adjustment, we want to seek the minimum

$$F(x) = e^r(x) g_{rs} e^s(x) \tag{2}$$

We know that a necessary condition for extreme value of a general continuous and differentiable multivariate function $\Phi(X)$ is $\nabla\Phi(\hat{X}) = 0$, it is to say that $\hat{X}$ is a stable point, however it is not sufficient. One of its sufficient conditions is that the Hessien matrix of $\Phi(\hat{X})$ is positive-define. The Taylor-series expansion of (2) evaluated at $\hat{X}$ is represented as $F(x) = F(\hat{x}) + F_a(\hat{x})\Delta x^a + \frac{1}{2} F_{\alpha\beta}(\hat{x})\Delta x^a \Delta x^\beta + \cdots$

We applied the above condition of multivariate function's extreme value to $F(x)$ [8], we have:

$$e^r(\hat{x}) g_{rs} A_a^s(\hat{x}) = 0 \tag{3a}$$

and

$$k_n \|e(\hat{x})\| < 1 \tag{3b}$$

where $A_a^r(\hat{x}) = \frac{\partial y^r}{\partial x^a}(\hat{x})$; $\Omega_{\alpha\beta}^r = \frac{\partial y^r}{\partial x^a \partial x^\beta}(\hat{x})$; $\Delta x^a = x^a - \hat{x}^a$; $K_n = \frac{\text{II}}{\text{I}}$;

$I = A_a^r(\hat{x}) g_{rs} A_\beta^s(\hat{x}) \Delta x^a \Delta x^\beta$ and $\text{II} = \frac{e^r(\hat{x})}{\|e(\hat{x})\|} g_{rs} \Omega_{\alpha\beta}^s \Delta x^a \Delta x^\beta$.

The formula (3) is the sufficient and necessary condition of seeking the extreme value of (2). It has a direct geometrical senses: formula (3a) means that the residual vectors are orthogonal to the tangent space at the extreme point; formula (3b) means that the extreme point lies in the circle whose center is y and whose radius is $\frac{1}{k_n}$.

2 Transformed surface and least-squares adjustment

Iterating method was often used in solving the nonlinear parametrical adjustment. To some extent, this method improves the accuracy of the adjusted results, but it is difficult to assess some characters of the resolution in the whole. However it show us an important inspiration from the progress of solving the model: the point which minimize the sum of squares derivations must be a stable point. In the view of differential geometry, the residual vector should be orthogonal to tangent surface at the stable point. Based above condition, we construct a

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hyper-surface, called Q-surface. This surface satisfies the following conditions:

a. It comes through the observational point Q.

b. A point in the Q-surface is generated by intersection of the normal plane and the line containing the point Q, and this line is orthogonal to the normal plane.

So the initial point moves along the model surface, the corresponding point in the hyper surface would vary accordingly. When the varied point in the hyper surface coincides with the point Q, the corresponding point $\bar{P}$ in the model surface is the adjusted point.

We would address the characters of the hyper-surface.

a. Construction of the Q-surface: See Fig. 1, based on the initial point P, the dimensionality of the tangent space of the model at the point P is m, and the dimensionality of the normal space is n-m. Because there are $n(n-m) = m$ lines which is orthogonal to the normal space through point Q in the n dimensional space, the hypersurface is also m dimensional manifold. For example, when the model surface is the two-dimensional manifold (sphere) embedded in three-dimensional space, the dimensionality of the tangent plane through random point P is two. There is one one-dimensional normal line through this point and there is one two-dimensional plane crossing the point Q that is orthogonal to the normal line. When the selected point P varies along the sphere, the track of the point P' is also a sphere whose radius is the line $0Q$.

b. The hyper-surface is also continuous and differentiable. We would describe it in details afterwards.

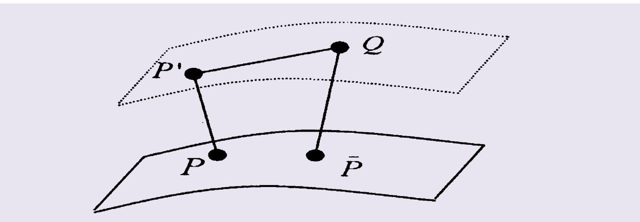

Fig. 1 Q-surface and parametric adjustment

3 The procedure of solving the adjust model

Through the above analysis we know that the hyper surface Q plays an important role in the adjustment. When the point P in the model surface moves along the model surface, the corresponding point P' in the Q-surface would varies. Conversely, When the point in the Q-surface moves along the Q-surface, the corresponding point P in the model surface would varies. So the vector $P\,\bar{P}$ and $P'Q$ would vary accordingly. See Fig. 2.

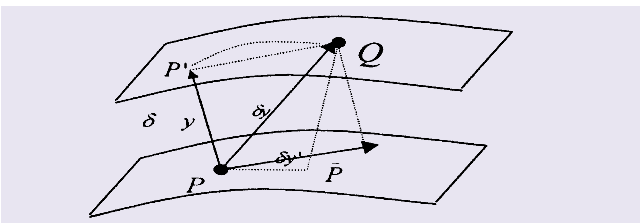

Fig. 2 Construction of Q-surface

We write $P\,\bar{P}$ as $\{\bar{y}'^{r} - y^{r}\} = \{\Delta y^{r}\}$ and $P'Q$ as $\{y^{r}_{Q} - y'^{r}\} = \{\Delta\tilde{y}^{r}\}$, where $\{y^{r}\}$ is the function of $\{y'^{r}\}$ and $\{\Delta y'^{r}\}$, which is related do $\{\Delta y^{r}\}$.

Let

$$\Delta y^{r} = \Delta y^{r}\left(\Delta y'^{s}\right) \tag{4}$$

So if we found out the obvious functional relation of formula (4), the direct method of solving adjustment model would appear. Followinglye would resolve it by constructing a pseudo polynomial.

Before commenting further, we introduce the notational conventions as follows

a. Convention of the upper/lower indices: the lower Roman letters $r, s, \ldots$ vary from 1 to n; the upper Roman letters $L, R, \ldots$ vary from m+1 to n; the lower Greek letters $\alpha, \beta, \ldots$ vary from 1 to m.

b. We express the coordinate of the points in the Q-surface as $\{\tilde{y}^{r}\}$ (point Q as $\{y^{r}_{Q}\}$) and the coordination of the points in model surface as $\{y^{r}\}$ (point $\bar{P}$ as $\{\bar{y}^{r}\}$). Write "Δ" before the coordination to indict the coordination difference of the points in the same surface and "δ" before the coordination to indict the coordination difference of the points in the different surface.

We solve the model (1) by four setps as following.

3.1 Three transformed expressions of the model surface [8]

Firstly, we write the equation of the model surface as

$$y^{r} = y^{r}\,(x^{a}) \tag{5}$$

Expand (5) into Taylor's series at initial value x^{a}_{0}

$$y^{r} = y^{r}(x^{a}_{0}) + A^{r}_{a}\Delta x^{a} + \frac{1}{2}\Omega^{r}_{\alpha\beta}\Delta x^{\alpha}\Delta x^{\beta} + \frac{1}{6}\Phi_{\alpha\beta\gamma}\Delta x^{\alpha}\Delta x^{\beta}\Delta x^{\gamma} + \cdots \tag{6}$$

where $\Delta x^{\alpha} = x^{\alpha} - x^{\alpha}_{0}$; $A^{r}_{\alpha} = \dfrac{\partial y^{r}}{\partial x^{\alpha}\partial x^{\beta}}(x^{\omega}_{0})$; $\Omega^{r}_{\alpha\beta} =$

$$= \frac{\partial^{2} y^{r}}{\partial x^{\alpha}\partial x^{\beta}}(x^{\omega}_{0}) \quad \text{and} \quad \Omega^{r}_{\alpha\beta\gamma} = \frac{\partial^{3} y^{r}}{\partial x^{\alpha}\partial x^{\beta}\partial x^{\gamma}}(x^{\omega}_{0})\,.$$

Formula (6) corresponds to the Gauss-form of the model surface.

In formula (6), a subset of m equations is divided a group and a subset of remaining n-m equations is divided into another group

$$\begin{cases} y^{\mu} = y^{\mu}(x^{\alpha}) & \text{(7a)} \\ y^{L} = y^{L}(x^{\alpha}) & \text{(7b)} \end{cases}$$

If $J = \left|\left(\dfrac{\partial y^{\mu}}{\partial x^{\alpha}}\right)_{m\times m}\right| \neq 0$, we can express $\{x^{\alpha}\}$ by $\{y^{\mu}\}$

$$x^{\alpha} = x^{\alpha}(y^{\mu}) = x^{\alpha}(y^{\mu}_{0}) + R^{\alpha}_{\lambda}\Delta y^{\lambda} + \frac{1}{2}\Lambda^{\alpha}_{\lambda\kappa}\Delta y^{\lambda}\Delta y^{\kappa} + + \frac{1}{6}\Theta^{\alpha}_{\lambda\kappa\delta}\Delta y^{\lambda}\Delta y^{\kappa}\Delta y^{\delta} + \cdots \tag{8a}$$

and

$$y^{L} = y^{L}(x^{\alpha}) = y^{L}(x^{\alpha}(y^{\mu})) = f^{L}(y^{\mu}) \tag{8b}$$

where $\Delta y^{\lambda} = y^{\lambda} - y_0^{\lambda}$; $R_{\lambda}^{\alpha} = \frac{\partial x^{\alpha}}{\partial y^{\lambda}}(y_0^{\omega})$; $\Lambda_{\lambda\kappa}^{\alpha} = \frac{\partial^2 x^{\alpha}}{\partial y^{\lambda}\partial y^{\kappa}}(y_0^{\omega})$ and $\Theta_{\lambda\kappa\delta}^{\alpha} = \frac{\partial^3 x^{\alpha}}{\partial y^{\lambda}\partial y^{\kappa}\partial y^{\delta}}(y_0^{\omega})$.

Formula (8) is equivalent to the Monge form of the model surface.

Substitute formula (8) for formula (7b), we can get

$$N^L = y^L - f^L(y^{\mu}) = 0 \tag{9}$$

which is the functional form of the model surface.

The expression of each order derivates and their relations would be derived.

Let $\bar{A}_{\alpha}^{r} = \{A_{\alpha}^{\rho}, A_{\alpha}^{R}\}$; $\Omega_{\alpha\beta}^{r} = \{\Omega_{\alpha\beta}^{\rho}, \Omega_{\alpha\beta}^{R}\}$ and $\Phi_{\alpha\beta\gamma}^{r} = \{\Phi_{\alpha\beta\gamma}^{\rho}, \Omega_{\alpha\beta\gamma}^{R}\}$.

The differentiation of the both sides of formula (9) with respect to $\{y^r\}$ is [8]

$$N_r^L = \frac{\partial N^L}{\partial y^r} = \{N_{\rho}^L, N_R^L\} = \left\{-\frac{\partial y^L}{\partial y^{\rho}}, \delta_R^L\right\} \tag{10}$$

The further differentiation of the both sides of formula (10) with respect to $\{y^r\}$ is

$$N_{\rho\mu}^L = \frac{\partial^2 N^L}{\partial y^{\rho}\partial y^{\mu}} = -\frac{\partial^2 y^L}{\partial y^{\rho}\partial y^{\mu}} \text{ and } N_{\rho\mu\tau}^L = \frac{\partial^3 N^L}{\partial y^{\rho}\partial y^{\mu}\partial y^{\tau}} = \frac{\partial^2 y^L}{\partial y^{\rho}\partial y^{\mu}\partial y^{\tau}} \tag{11}$$

If we adopt $\{y^a\}$ as the coordination of the model surface, the Gauss-form of the model surface is written as $y^r = h^r(y^a)$ and the partial derivates with respect to new coordination system are listed as follows

$$\frac{\partial y^r}{\partial y^{\alpha}} = \left\{\frac{\partial y^{\rho}}{\partial y^{\alpha}}, \frac{\partial y^R}{\partial y^{\alpha}}\right\} = \{\delta_{\alpha}^{\rho}, -N_{\alpha}^R\} \tag{12a}$$

$$\frac{\partial^2 y^r}{\partial y^{\alpha}\partial y^{\beta}} = \{0, -N_{\alpha\beta}^R\} \tag{12b}$$

$$\frac{\partial^3 y^r}{\partial y^{\alpha}\partial y^{\beta}\partial y^{\gamma}} = \{0, -N_{\alpha\beta\gamma}^R\} \tag{12c}$$

Due to the deriving rules of the compound function that is

$$\frac{\partial y^r}{\partial y^{\kappa}} = \frac{\partial y^r}{\partial x^{\lambda}} \cdot \frac{\partial y^{\lambda}}{\partial y^{\kappa}} \tag{13}$$

Substitute formula (13) with the above outcomes $\{\delta_{\kappa}^{\rho}, -N_{\kappa}^{R}\} = \{A_{\lambda}^{\rho}, A_{\lambda}^{R}\} R_{\kappa}^{\lambda}$, then

$$A_{\lambda}^{\alpha} R_{\kappa}^{\lambda} = \delta_{\kappa}^{\alpha} \tag{14a}$$

and

$$N_{\kappa}^L = A_{\lambda}^L R_{\kappa}^{\lambda} \tag{14b}$$

The further differentiations of formula (13) are

$$\Lambda_{\kappa\tau}^{\lambda} = -R_{\mu}^{\lambda}\Omega_{\beta\gamma}^{\mu}R_{\kappa}^{\beta}R_{\tau}^{\gamma} \tag{15a}$$

$$N_{\kappa\tau}^L = -(N_{\mu}^L\Omega_{\beta\gamma}^{\mu} + \Omega_{\beta\gamma}^L)R_{\kappa}^{\beta}R_{\tau}^{\gamma} \tag{15b}$$

Similarly, one can obtain $\Theta_{\kappa\tau\omega}^{\lambda}$ and $N_{\kappa\tau\omega}^L$.

3.2 Computing the $\{\tilde{y}^r\}$

As we know that $\{\delta\tilde{y}\} = \{\tilde{y} - y\}$ yis orthogonal to the model surface, it must be expressible as a linear combination of the *n-m* gradient-vectors

$$\delta\tilde{y}_s = N_s^M C_M \tag{16}$$

On the other hand, $\{\delta y'\}$ is orthogonal to gradient vector

$$N_r^L \delta_{y'^r} = 0 \tag{17}$$

Substitute formula (17) with $\delta_{y'} = \delta y - \delta_{\tilde{y}}$ and $\delta_{\tilde{y}^r} = g^{rs}\delta_{\tilde{y}_s}$, then we can get $N_r^L g^{rs} N_s^M C_M = N_r^L \delta y^r$.

The resolutions are as follows

$$C_K = Q_{KL}N_m^L\delta y^m \tag{18a}$$

$$\delta\tilde{y}^r = g^{rs}N_s^K C_K \tag{18b}$$

where $Q_{KL}B^{LM} = \delta_K^M$ and $B^{LM} = N_r^L g^{rs} N_s^M$.

3.3 The construction of pseudo-multinomial

From Fig. 2, we can find $\delta y^r = y_Q^r - y^r$ and $\delta\tilde{y}^r = \tilde{y}^r - y^r$, among which we select m variables that can be written as $\delta y^{\alpha} = y_Q^{\alpha} - y^{\alpha}$ and $\delta\tilde{y}^{\alpha} = \tilde{y}^{\alpha} - y^{\alpha}$. In order to construct the pseudo-multinomial we can prove that $\{\tilde{y}^{\alpha}\}$ is the function of $\{y^{\alpha}\}$.

Let $\Delta y^{\alpha} = \tilde{y}^{\alpha} - y^{\alpha}$ and $\Delta\tilde{y}^{\alpha} = y_Q^{\alpha} - \tilde{y}^{\alpha}$, then $\{\Delta\tilde{y}^{\alpha}\}$ must be the function of $\{\Delta\tilde{y}^{\alpha}\}$. Again let

$$\tilde{A}_{\beta}^{\alpha} = \frac{\partial\tilde{y}^{\alpha}}{\partial y^{\beta}} = \delta_{\beta}^{\alpha} + \frac{\partial\delta\tilde{y}^{\alpha}}{\partial y^{\beta}} \tag{19a}$$

$$\tilde{\Omega}_{\beta\gamma}^{\alpha} = \frac{\partial^2\tilde{y}^{\alpha}}{\partial y^{\beta}\partial y^{\gamma}} = \frac{\partial^2\delta\tilde{y}^{\alpha}}{\partial y^{\beta}\partial y^{\beta}} \tag{19b}$$

Then we can get

$$\frac{\partial\Delta\tilde{y}^{\alpha}}{\partial y^{\beta}} = \frac{\partial\left(y_Q^{\alpha} - \tilde{y}^{\alpha}\right)}{\partial y^{\beta}} = -\tilde{A}_{\beta}^{\alpha} \quad \frac{\partial^2\Delta\tilde{y}^{\alpha}}{\partial y^{\beta}\partial y^{\gamma}} = -\tilde{\Omega}_{\beta\gamma}^{\alpha} \tag{20}$$

We must calculate the inverse partial derivative to obtain $\{\Delta y^{\alpha}\}$ which is a function of $\{\Delta\tilde{y}^{\alpha}\}$. Let $\tilde{R}_{\beta}^{\alpha} = \frac{\partial y^{\alpha}}{\partial\tilde{y}^{\beta}}$, $\tilde{\Lambda}_{\beta\gamma}^{\alpha} = \frac{\partial^2\tilde{y}^{\alpha}}{\partial y^{\beta}\partial y^{\gamma}}$.

The samilar as the conclusion of formulae (14) and (15), we can get the following formulae

$$\tilde{A}_{\beta}^{\alpha}\tilde{R}_{\kappa}^{\beta} = \delta_{\kappa}^{\alpha} \tag{21a}$$

and

$$\tilde{\Lambda}_{\beta\gamma}^{\alpha} = -\tilde{R}_{\mu}^{\alpha}\tilde{\Omega}_{\eta\tau}^{\mu}\tilde{R}_{\beta}^{\eta}\tilde{R}_{\gamma}^{\tau} \tag{21b}$$

From the formula (21), we find that we can calculate the partial derivative of all powers if we know $\widetilde{A}^{\alpha}_{\gamma}$ and $\widetilde{\Omega}^{\mu}_{\eta\tau}$. The following is the formula to calculate them

$$\widetilde{A}^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} + T^{\alpha\mu} E_{\mu\beta} + G^{\alpha}_{L} U^{L}_{\beta} \tag{22}$$

where $T^{\alpha\mu} = g^{\alpha\mu} - G^{\alpha}_{\kappa} P^{\mu\kappa}$; $E_{\mu\beta} = C_{\kappa} N^{\kappa}_{\mu\beta}$; $G^{\alpha}_{L} = P^{\alpha\kappa} Q_{KL}$; $U^{L}_{\beta} = N^{L}_{\beta\mu} \Delta\widetilde{y}^{\mu}$; $P^{\alpha\kappa} = g^{\alpha s} N^{K}_{s}$;

$C_{K} = Q_{KL} W^{L}$; $W^{L} = N^{L}_{\mu} \delta y_{\mu}$

$B^{MN} Q_{NK} = \delta^{M}_{K}$, $B^{KM} = N^{K}_{\gamma} g^{rs} N^{M}_{s} = N^{K}_{\gamma} g^{\gamma\delta} N^{M}_{\delta} + g^{K\delta} N^{M}_{\delta} + g^{\delta M} N^{K}_{\delta} + g^{KM}$.

With the same method we can get $\widetilde{\Omega}^{\alpha}_{\beta\gamma}$. Therefore $\{\Delta y^{\alpha}\}$ can be expanded into Taylor's series

$$\Delta y^{\alpha} = \widetilde{R}^{\alpha}_{\beta} \Delta\widetilde{y}^{\beta} + \frac{1}{2} \widetilde{\Lambda}^{\alpha}_{\beta\gamma} \Delta\widetilde{y}^{\beta} \Delta\widetilde{y}^{\gamma} + \ldots \tag{23}$$

whose linear items have not only the first partial derivative but also the second partial derivative, and the second power items include not only the second partial derivative but also the third partial derivative. The formula can improve the accuracy of adusted values.

3.4 the calculation of adjusted values and parameters and the evaluation of their accuracy

After the calculation of $\{\Delta y^{\alpha}\}$, we can get the parameters based on the transformed relation between $\{\Delta y^{\alpha}\}$ and $\{\Delta x^{\beta}\}$, that is

$$\Delta x^{\alpha} = R^{\alpha}_{\beta} \Delta y^{\beta} + \frac{1}{2} \Lambda^{\alpha}_{\beta\gamma} \Delta y^{\beta} \Delta y^{\gamma} + \frac{1}{6} \Theta^{\alpha}_{\beta\gamma\delta} \Delta y^{\beta} \Delta y^{\gamma} \Delta y^{\delta} + \ldots \tag{24a}$$

The other *n*-*m* adjustments can be get with the Monge's model surface

$$\Delta y^{L} = A^{L}_{\beta} \Delta x^{\beta} + \frac{1}{2} \Omega^{L}_{\beta\gamma} \Delta x^{\beta} \Delta x^{\gamma} + \frac{1}{6} \Phi^{L}_{\beta\gamma\delta} \Delta y^{\beta} \Delta y^{\gamma} \Delta y^{\delta} + \ldots \tag{24b}$$

From the formula (24) we can get the parameter values and the adjusted values of the corresponding adjusted point $\bar{P}$

$$\bar{x}^{\alpha} = x^{\alpha} + \Delta x^{\alpha} \tag{25a}$$

$$\bar{y}^{r} = x^{r} + \Delta x^{r} \tag{25b}$$

In the meantime we can calculate the mean square error of unit weight

$$m_0 = \pm\sqrt{\frac{\delta\bar{y}^{r} g_{rs} \delta\bar{y}^{s}}{n-m}} = \pm\sqrt{\frac{\left(y^{r}_{Q} - \bar{y}^{r}\right) g_{rs} \left(y^{s}_{Q} - \bar{y}^{s}\right)}{n-m}} \tag{26}$$

The co-variance of adjusted parameters is

$$\bar{\alpha}^{\alpha\beta} = R^{\alpha}_{r}(\hat{x}) g^{rs} R^{\beta}_{s}(\hat{x}) \tag{27}$$

The other adjustments can also be calculated with the same above method.

4 The analysis on the case

There are error equations

$$\begin{cases} l^1 = \cos x^1 + v^1 \\ l^2 = \cos x^1 + v^2 \end{cases}$$

where l^1 and l^2 are measurements which are not relative, $l^1 \sim N(2, 1)$ and $l^2 \sim N(2, 1)$; x^1 is the observing parameter whose initial value is $x^1_0 = \frac{\pi}{3}$. Then the equation of the model face is

$$\begin{cases} l^1 = \cos x^1 \\ l^2 = \cos x^1 \end{cases}$$

From the method showed in the above, we can calculate by four steps

Step 1: to calculate the partial derivative of all powers.

$(A^1_1, A^2_1) = (-0.866, 0.5)$, $R^1_1 = -1.155$, $N^2_1 = 0.577$.

$(\Omega^1_{11}, \Omega^2_{11}) = (-0.5, 0.866)$, $\Lambda^1_{11} = -0.770$, $N^2_{11} = 1.540$.

$(\Phi^1_{111}, \Phi^2_{111}) = (-0.866, 0.5)$ $\Theta^1_{111} = -3.080$, $N^2_{111} = 3.080$.

$N^2_2 = 1$, $N^2_{12} = N^2_{21} = N^2_{22} = 0$.

Step 2: to calculate $\delta\bar{y}^1$.

$B^{22} = 1.333$, $Q_{22} = 0.750$, $C_2 = 1.50$, $\delta\bar{y}^1 = 0.866$, $\bar{y}^1 = 1.366$.

Step 3: to calculate the coefficiences of the pseudo-multinomial.

$\widetilde{A}^1_1 = 2.536$, $\widetilde{R}^1_1 = 0.394$.

Step 4: to calculate all adjusted vlues.

$\Delta y^1 = \widetilde{y}^1 - y^1 = 0.250$, $\bar{y}^1 = 0.750$.

$\Delta x^1 = \bar{x}^1 - x^1 = -0.321$, $\bar{x}^1 = 0.726$.

$\Delta y^2 = \widetilde{y}^2 - y^1 = 0.202$, $\bar{y}^2 = 0.664$.

$m^0 = \pm 1.830$.

The method is compared with the linear method, the result is listed in the Tab. 1.

Tab. 1 the comparation between the pseudo-multinomial method and the linear method

method	$\bar{x}^1$	$\bar{y}^1$	$\bar{y}^2$	m^0
The linear method	0.415	0.915	0.403	1.931
The pseudo-multinomial method	0.726	0.750	0.664	1.830

This paper first discusses the condition that the sum of square errors must minimize and its geometrical characteristics. The paper uses a super curved surface that is a transformed surface. The construction of the surface and its features are discussed in detail. The paper first puts forward a fitting method of pseudo-multinomial to solve the nonlinear adjustment and gives the accuracy evaluation of the adjusted model. Finally, a simple case indicates the model is practical.

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Abstract

First, this paper gives the optimal condition and its geometrical characters of the least-square adjustment. Then the relation between the transformed surface and least squares is addressed. Based on the above, a non-iterate method, called the fitting method of pseudo-polynomial, is derived in detail. The final least-squares solution can be determined with sufficient accuracy in a single step and is not attained by moving the initiative point in the view of iteration. The accuracy of the solution relys wholly on the frequency of Taylor's series. The example verifies the correctness and validness of the method.

Topografische Aufnahmen in der Antarktis

Mehr als 500 000 km^2 nahm der russische staatliche Betrieb Aerogeodezija vom sechsten Kontinent topografisch auf. Die Luftbildaufnahmen wurden mit Luftbildkammern AFA TE-50 und TE-100 sowie Flugzeugen IL-14 in den Maßstäben 1:4000 und 1:200 000, in der Regel mit 60 % Längs- und 40 % Querüberdeckung ausgeführt. Dabei wurden Statoskopangaben, Funkhöhenmesser RVTV und funkgeodätische Stationen RDS verwendet. Die Lagekoordinaten der Aufnahmezentren wurden mit einem Programm des Zentralen Forschungsinstituts (CNIIGAiK) berechnet. Sie dienten in Verbindung mit den Höhen als Ausgangsdaten für die Verdichtung mit Hilfe der analytischen räumlichen Bildtriangulation. Die Netze wurden mit Fotoreduktoren reduziert.

Bildpläne sowie ihre Montagen in 1:50 000 und 1:100 000 entstanden mit Hilfe des Entzerrungsgeräts SEG V, die Reliefzeichnung mit topografischen Stereometren STD-2 oder Universalgeräten SD und SPR.

Die Kartenoriginale wurden von ebenen Gebieten an Hand der Bildpläne, von bergigen Gebieten mit Hilfe einfacher Projektoren in 1:50 000 und 1:100 000 gefertigt, die Herstellungsoriginale in drei Farben ausgeführt. Die Karten wurden im Gauss-Krüger-System vorbereitet. Als Höhensystem wurde das Niveau des südlichen Ozeans angehalten. Alle Karten entstanden mittels Gravur.

Seit 1970 erschienen 80 Blätter im Maßstab 1:100 000 und 100 Blätter im Maßstab 1:200 000. Diese topografischen Karten der Antarktis fanden wegen ihres Informationsgehalts und ihrer Genauigkeit internationale Anerkennung.

Seit 1985 wurden Spezialkarten der Antarktis, u. a. mit Hilfe des Funkortungssystems RLS und der Flugzeuge IL-14 und IL-18 D in 1:500 000 und 1:1 000 000 hergestellt. Das Relief und die Dicke des Eises wurden mit dem Impulsverfahren ermittelt. Mit Hilfe von Luftbildkammern AFA TE-10 entstanden Bildpläne 1:200 000 und 1:1 000 000. Die mittleren Lagefehler betrugen 104 m mit IL-14 bzw. 124 m mit IL-18 D. Zum Höhenanschluss dienten Aero- und barometrische Nivellements mit Lagefehlern von 6,6 bzw. 11,8 m.

Seit 1975 wurden auch großmaßstäbige Aufnahmen 1:2000 von 25 km^2 und 1:10 000 von 12 km^2 der Antarktisstationen Bellinghausen, Mirny, Molodežnaja, Novaja Lazarevskaja und Russkaja mit dem Messtisch ausgeführt. Die Punkte wurden mit Theodolitzügen (mit Sekundentheodoliten 2T2 und elektrooptischen Streckenmessgeräten NOK 2000 von Carl Zeiss JENA) festgelegt.

Aus: Topografičeskie s-emki v Antarktide. Von Juskevič A. V. – Geodez. i Kartogr., Moskva (2000) 6, S. 12–16